

Barrier Options

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QF 301

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How to Cheapen an Option...

- Change the underlying (lower volatility, higher dividend yield)
- Change the strike, or time to maturity
- Vertical Spreads
- Knock-out or Knock-in options

Barrier Options

- Payoff depends on not just the terminal level of underlying asset but also its **path**.
- Both the option Strike and **Barrier** are going to dictate the payoff.
- Options can be **knock-in** or **knock-out**
- Barrier can be related to the underlying asset (inside barrier) or another asset (outside barrier)
- European Knock In Option+ European Knock Out Option = corresponding Vanilla European Option

Barrier Options (contd.)

- Investors can express more complex views than the simple bullish or bearish scenarios that a Vanilla call will permit.
- Barrier options will be generally cheaper.
- Often, the sensitivity of barrier options to market will be very sharp, bigger than that of vanilla options.
- Cheap for a reason - hedge may vanish when most needed.

Absconding Hedge!!!



Barrier and Vanilla Options Compared

Strike = 100

T = 0.5

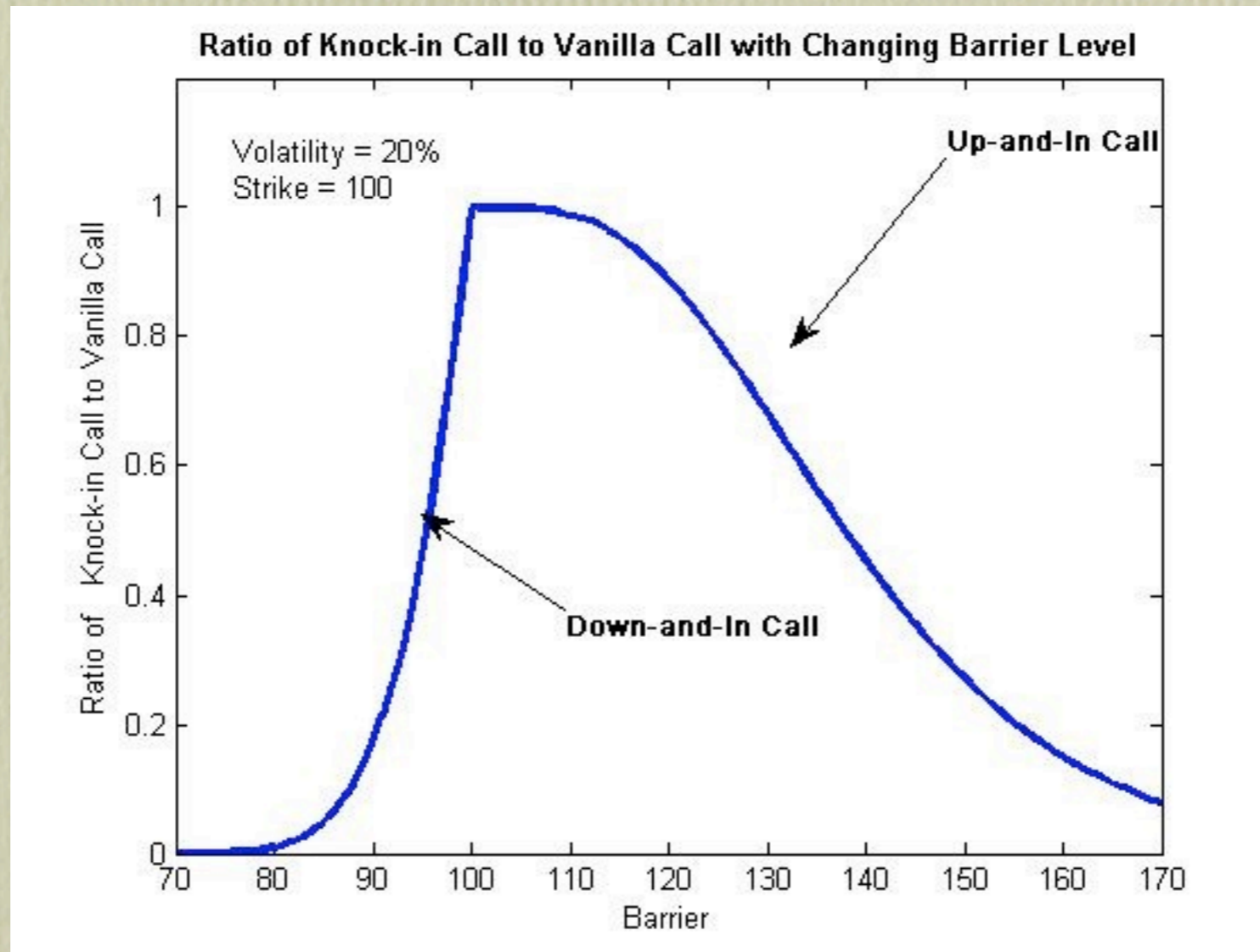
Volatility = 25%

r = 0

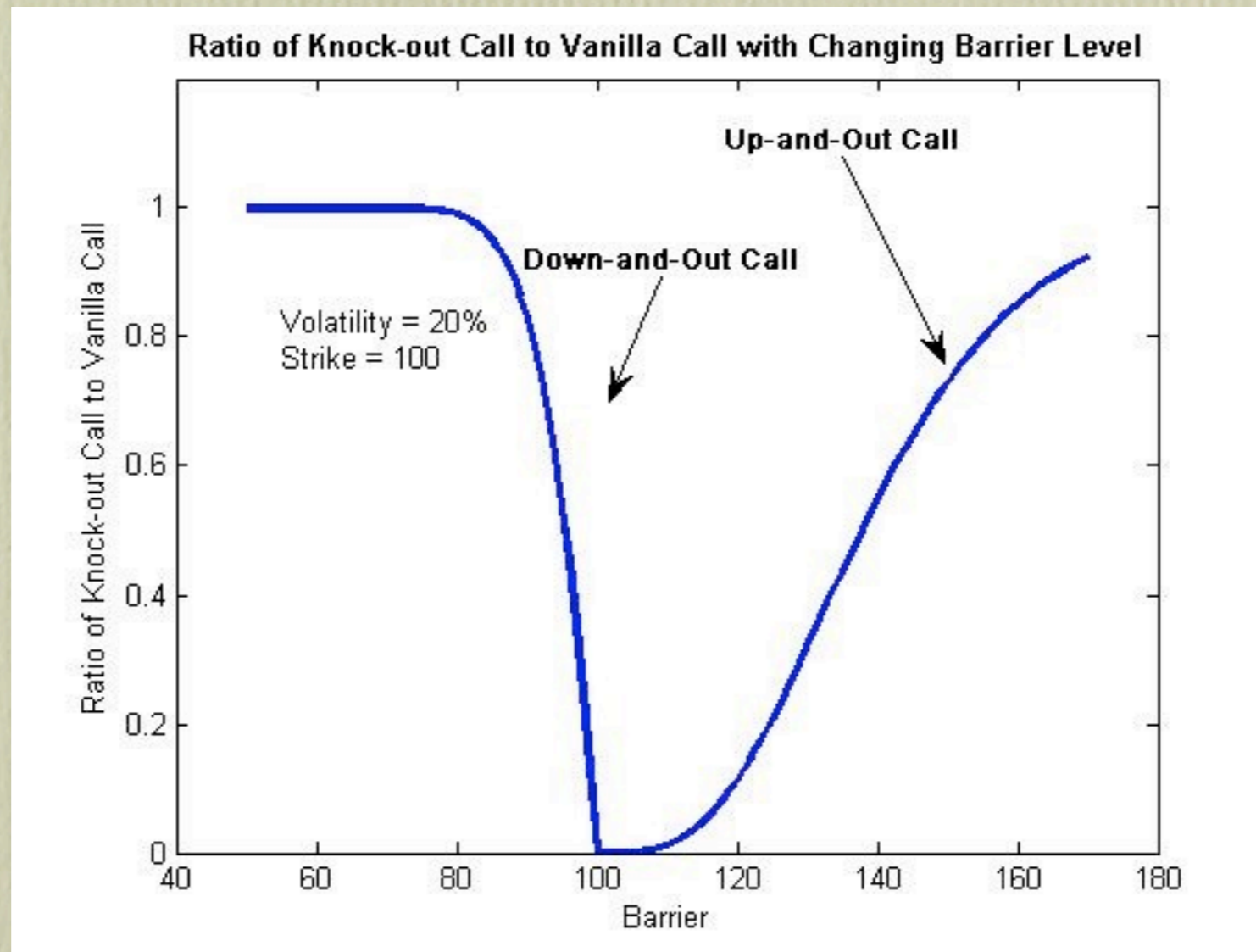
q = 0

S	BARRIER						
	70	80	90	100	110	120	-
Vanilla Call	0.12	0.78	2.84	7.04	13.44	21.52	
Down and Out Call	0	0	2.67	7.01	13.44	21.51	80
Down and In Call	0.12	0.78	0.17	0.03	0.01	0	80
Up and Out Call	0.08	0.42	1.03	1.41	1.02	0	120
Up and In Call	0.04	0.36	1.81	5.64	12.42	21.52	120
Vanilla Put	30.12	20.78	12.84	7.04	3.44	1.52	
Down and Out Put	0	0	1.97	2.4	1.76	0.98	80
Down and In Put	30.12	20.78	10.87	4.65	1.68	0.53	80
Up and Out Put	30.12	20.78	12.82	6.92	2.93	0	120
Up and In Put	0	0	0.02	0.13	0.51	1.52	120

Knock-In Call Compared to Vanilla Call



Knock-out Call Compared to Vanilla Call



Barrier Option Greeks

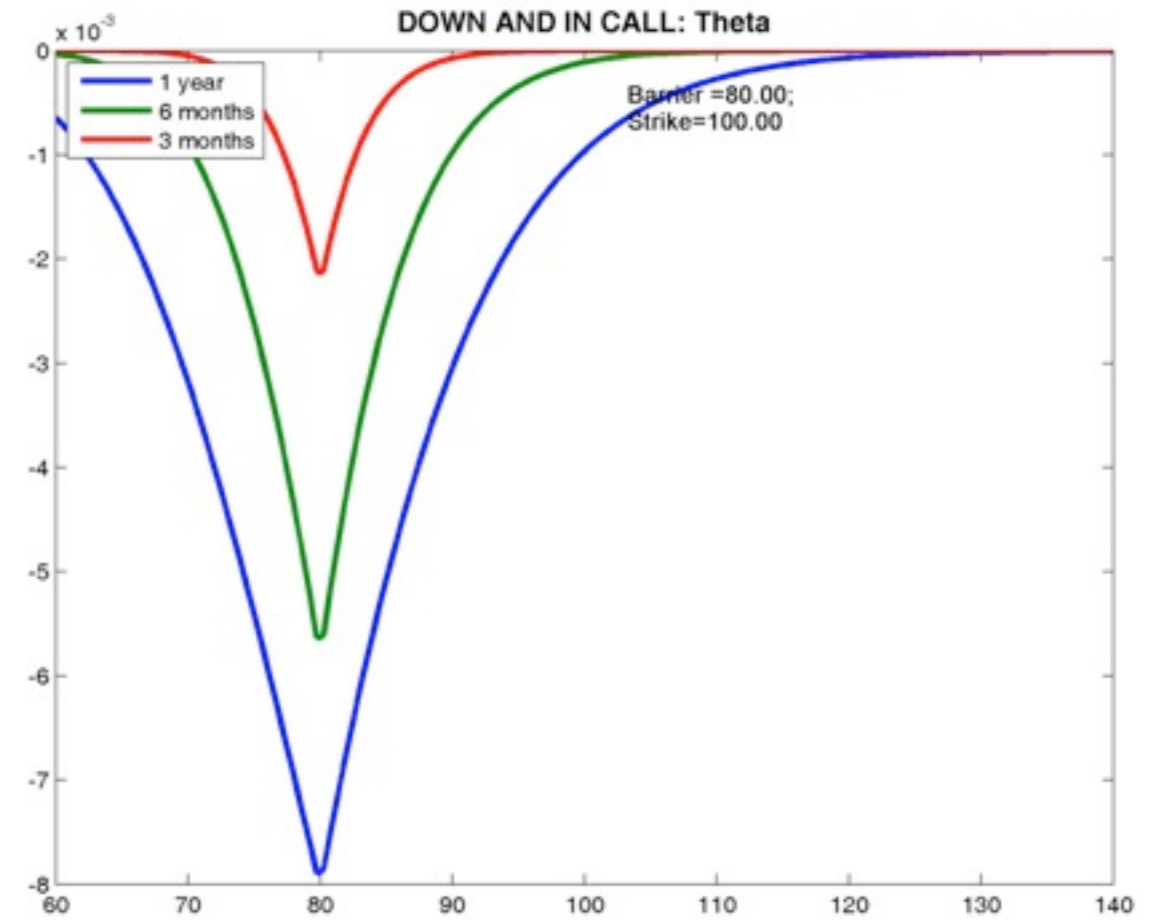
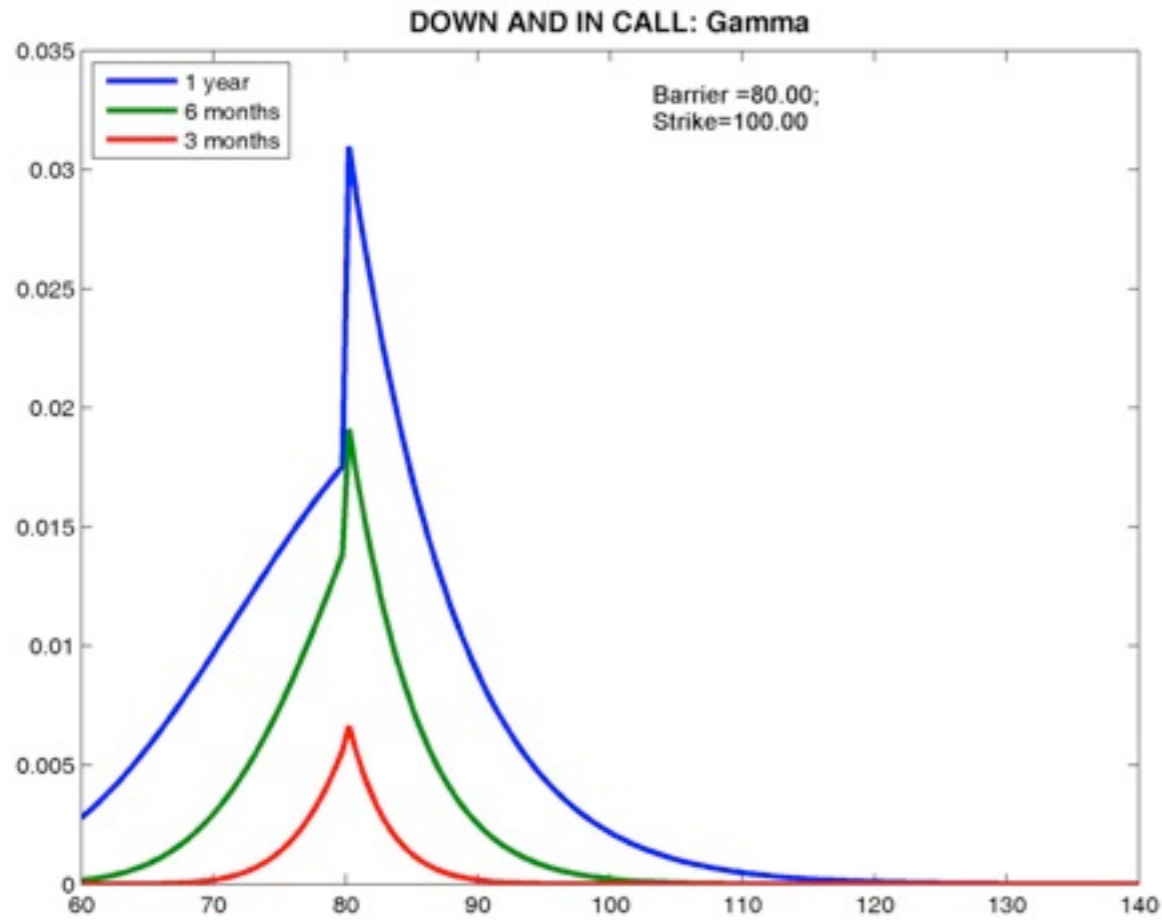
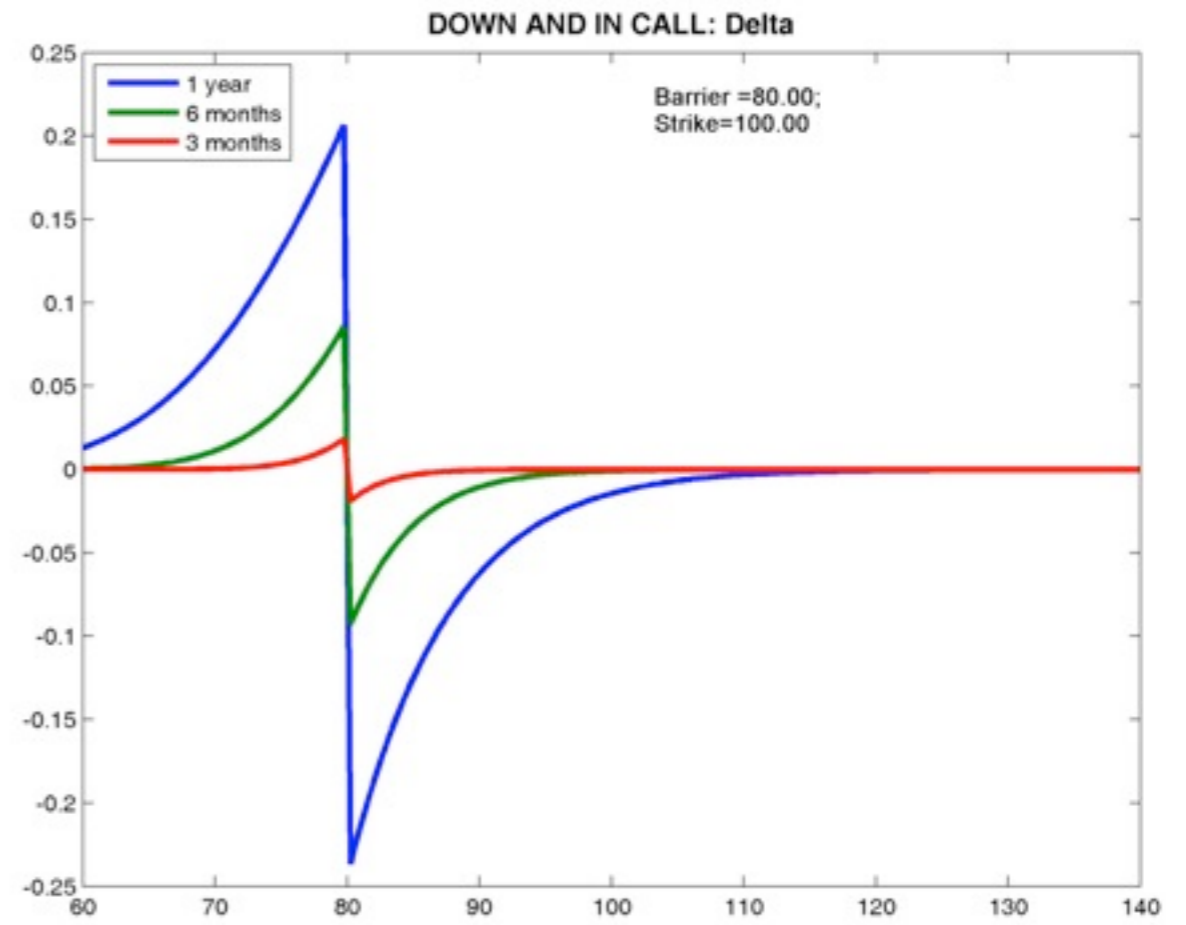
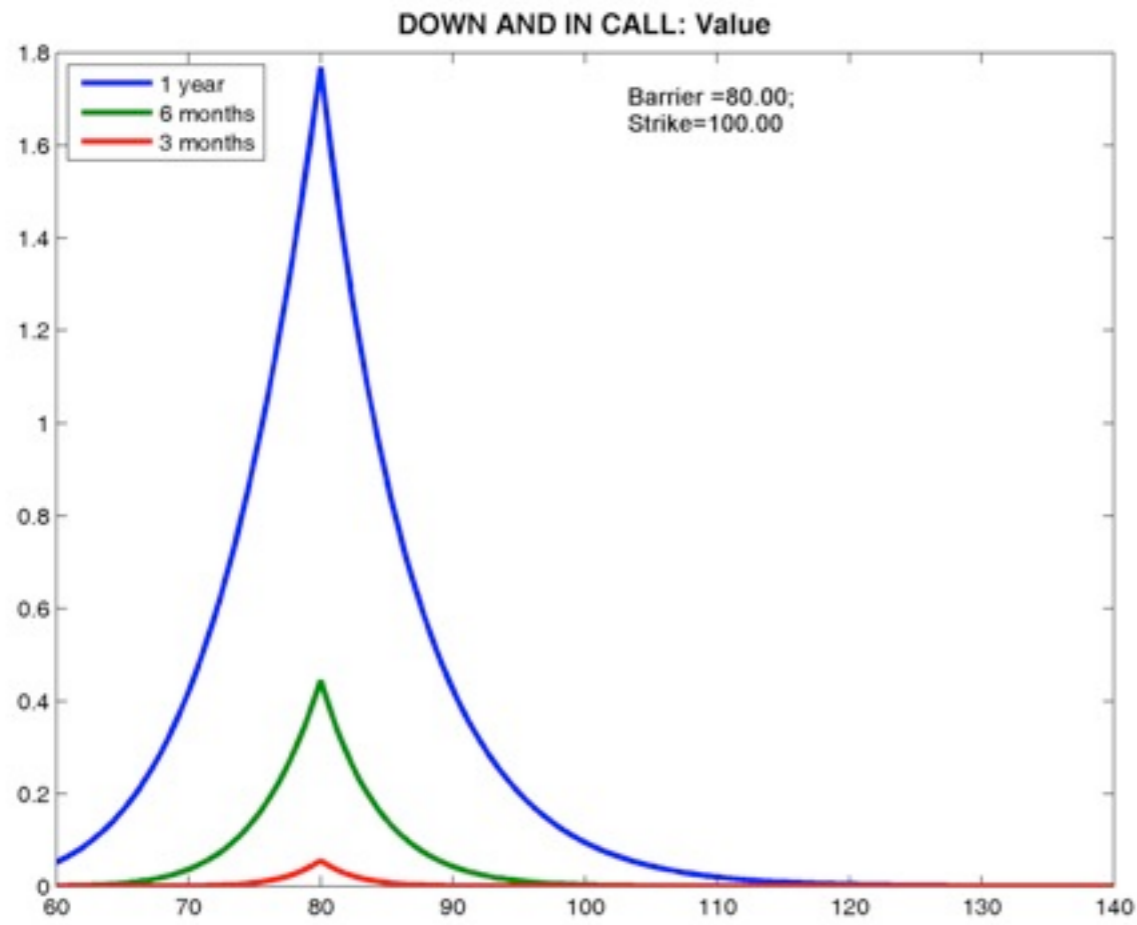
- Risk Management of Barrier options is very difficult
- Especially when the underlying is near the barrier
- And even more so when there is very little time left ...
- One can use dynamic hedges (delta hedging) or static hedges (use other options).
- Gamma is very large exactly at the barrier.
- The second region where gamma is large is near but slightly away from barrier where its major

Barrier Options: A Few Examples

- An up-and-out call is cheaper than the standard call. As the underlying moves up the vanilla call will always gain in value but the barrier option will initially gain but then start to decline as the risk of getting knocked out will increase.
- There are two competing effects - increase in value of the payoff, and the decrease in probability of receiving it. This makes the delta fluctuate and it can switch from positive to negative.

Down and In Call

- When the price of the underlying stock is high, the Down and In Call can be worth a lot less than the Vanilla Call if barrier is far, because it will get knocked in only when the stock has fallen a lot. Below the barrier, it is the same as vanilla call.
- Value greatest at the barrier!
- Just below barrier, delta is the same as that of vanilla call. Just above barrier, delta is negative - as stock price increases, the knock-in probability decreases. Big change from positive to negative delta at barrier means delta hedging is difficult (long to short stock as barrier crossed). Gamma always positive and very large at barrier.

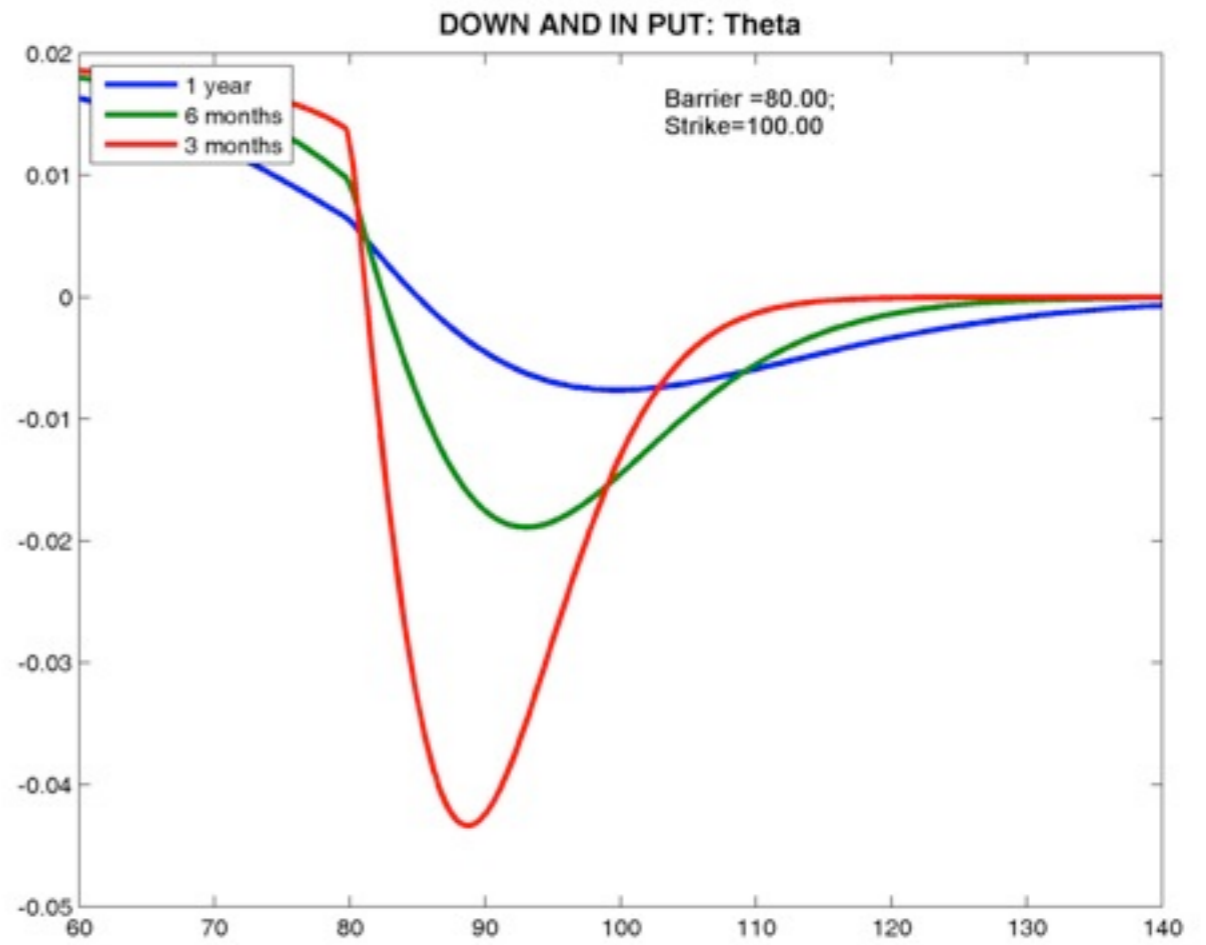
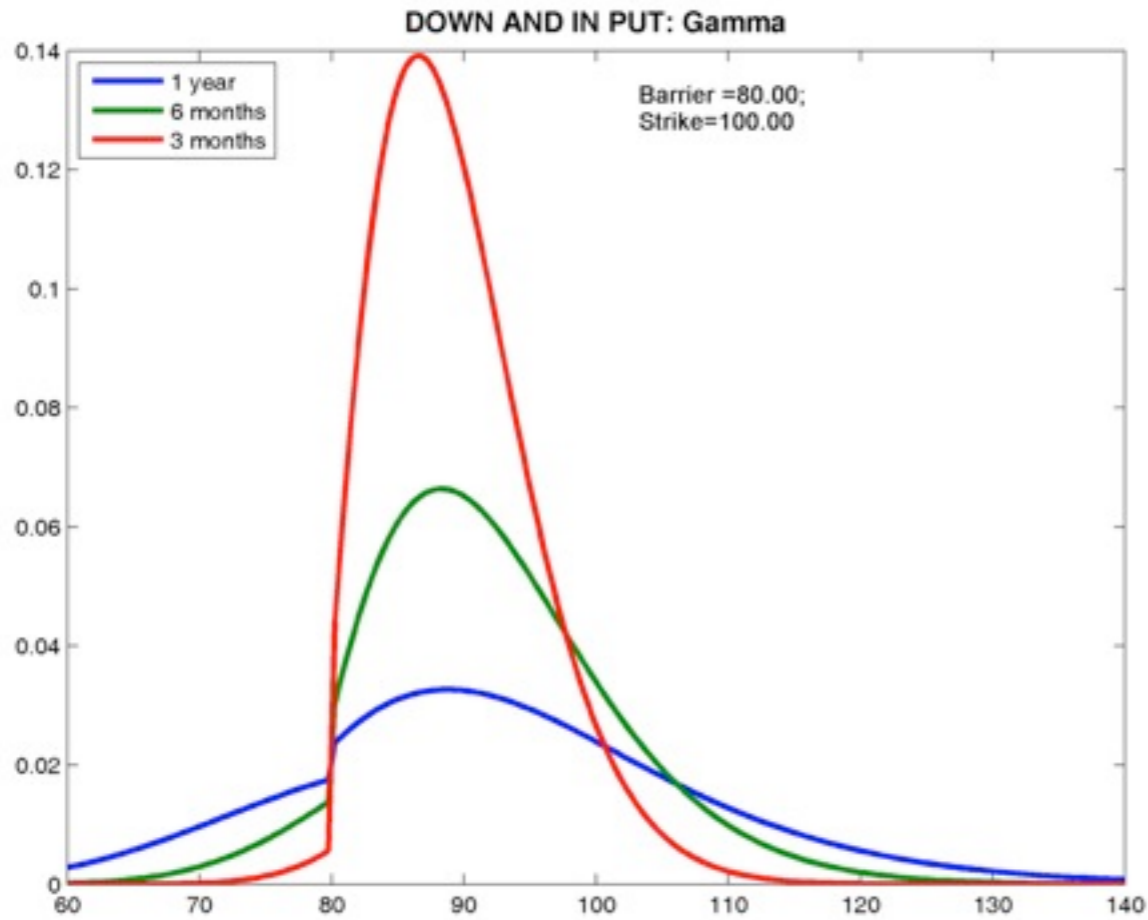
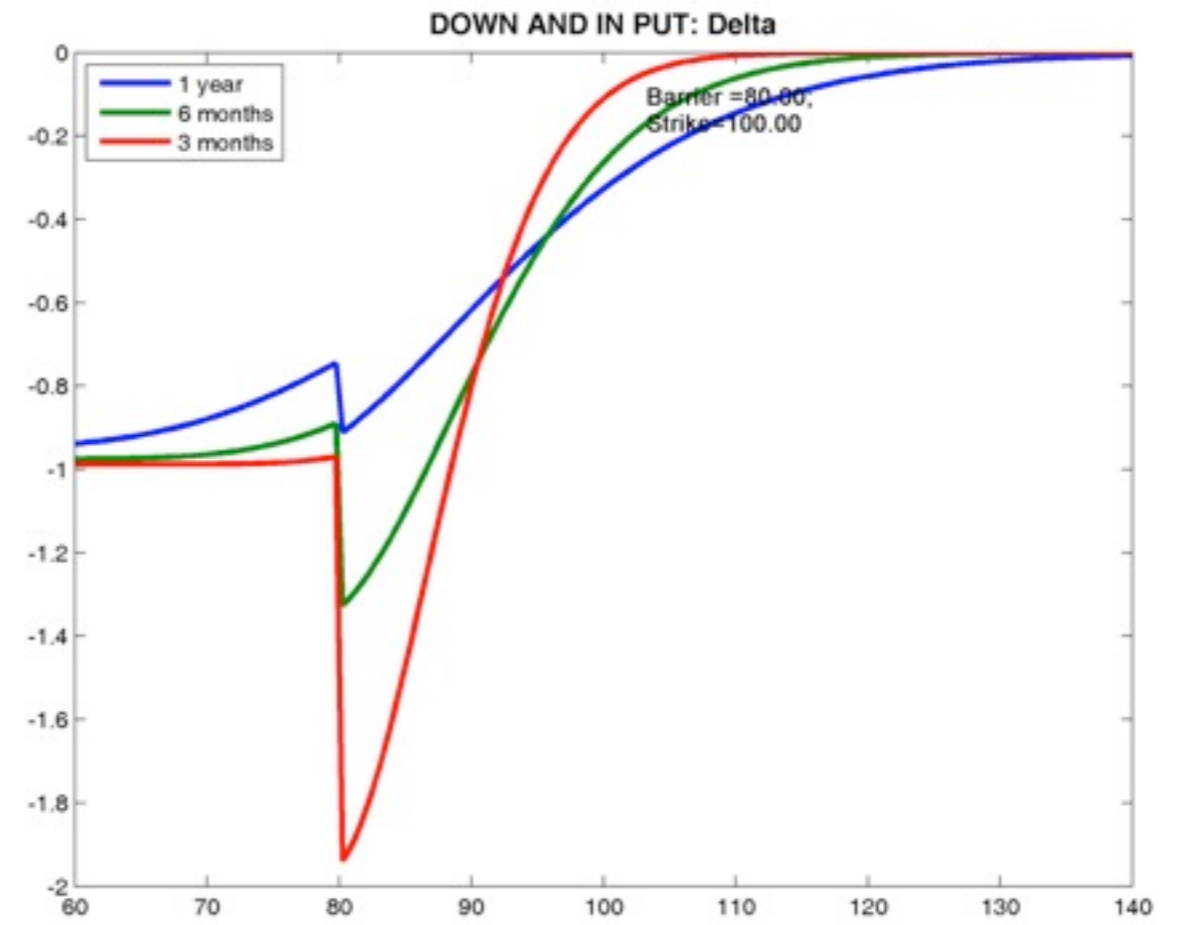
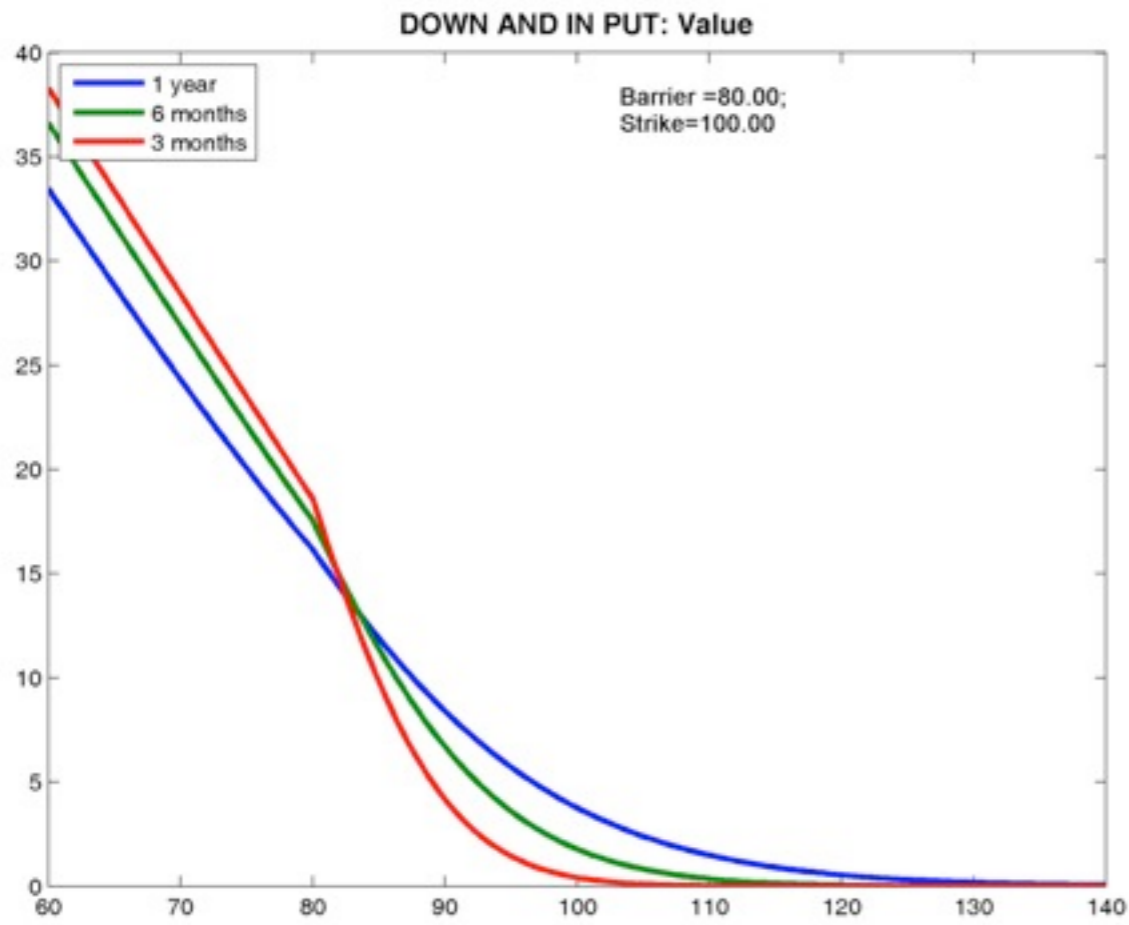


Down and In Put

Since much of the value of a standard put is due to downward moves, which would also cause the Down and In Put to be knocked in, both options can have similar value. If strike is below the barrier, it is worth the same as Vanilla put.

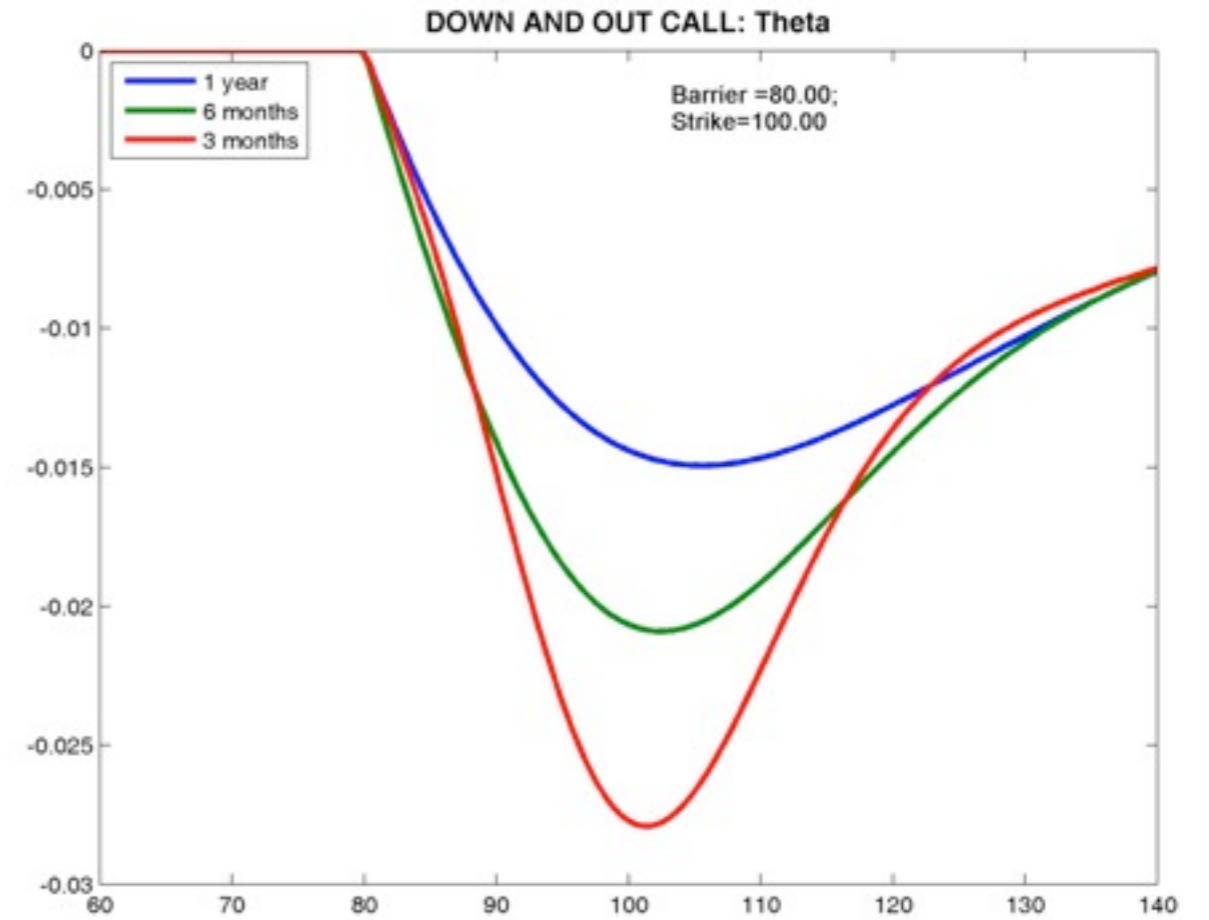
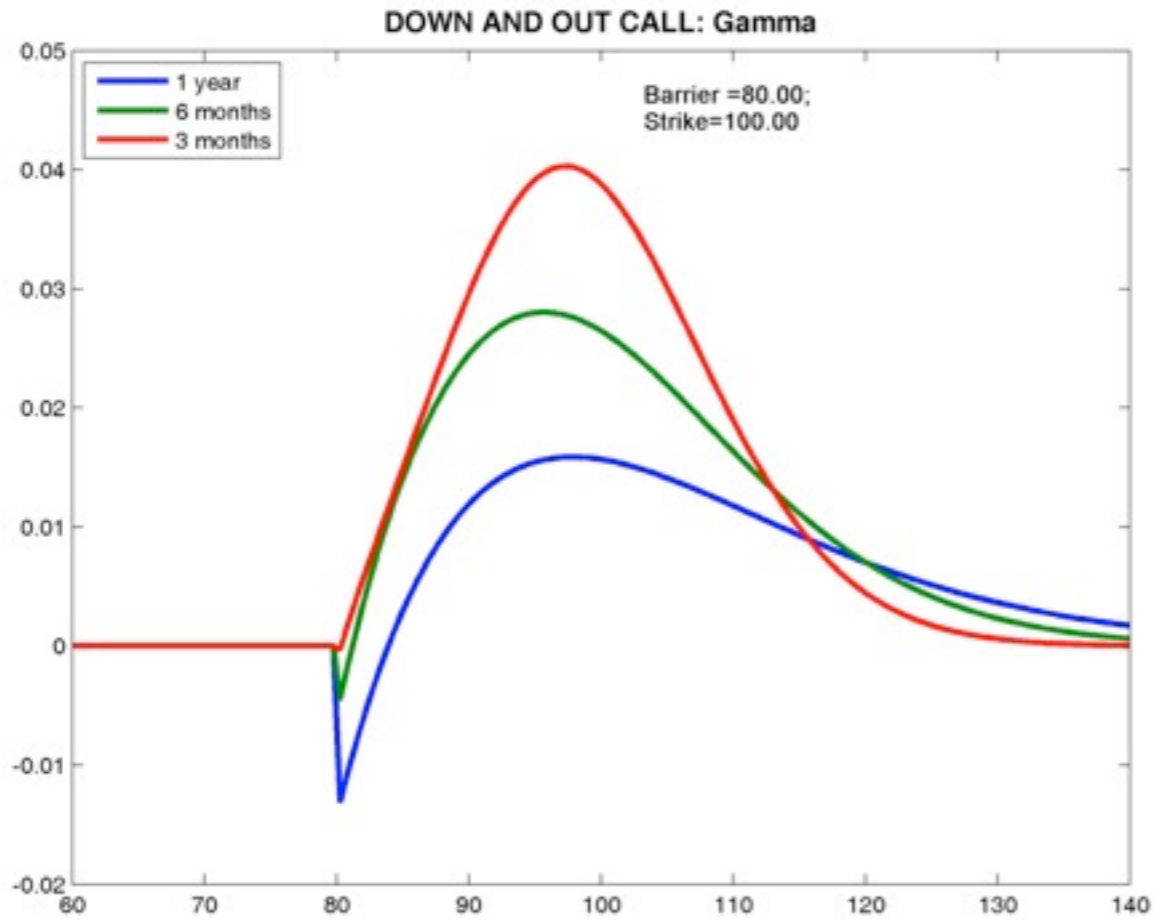
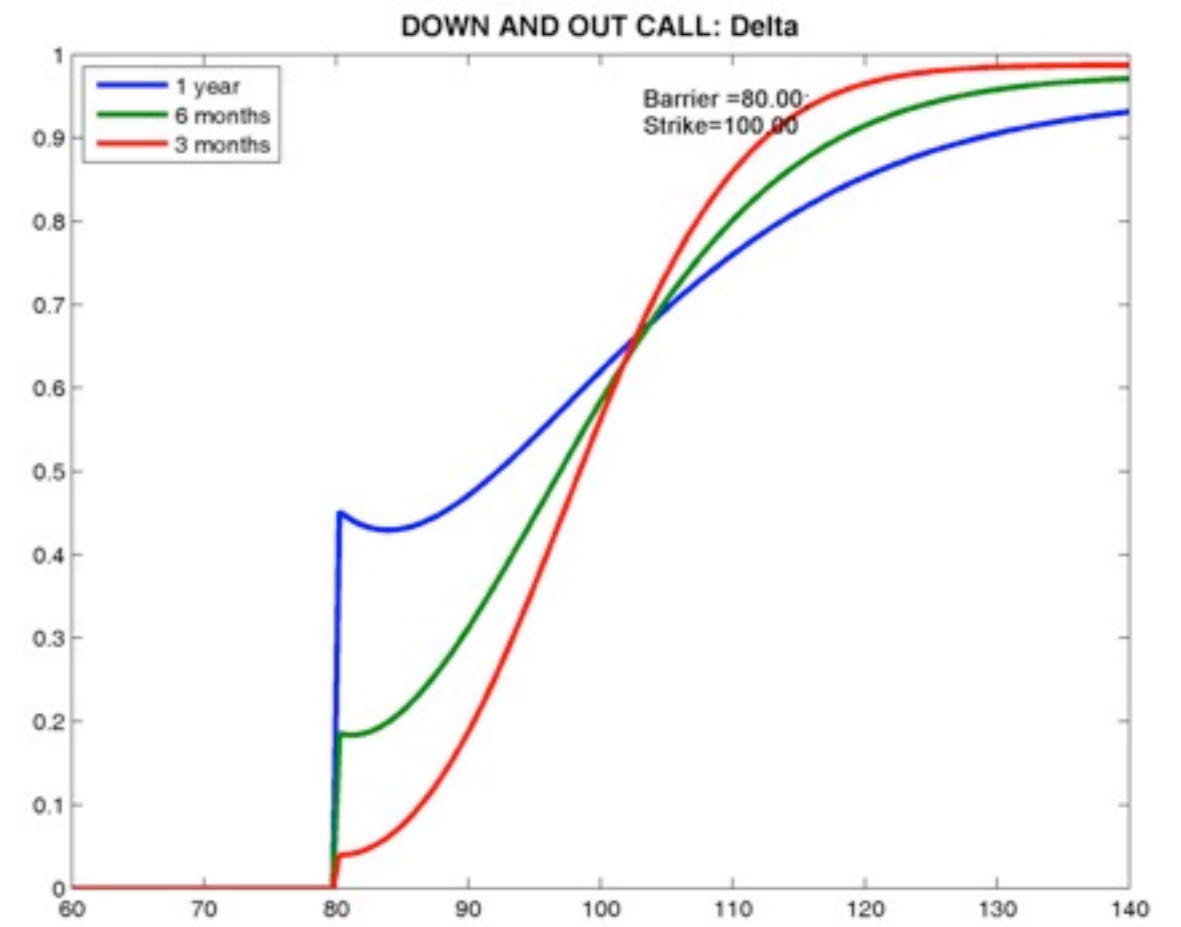
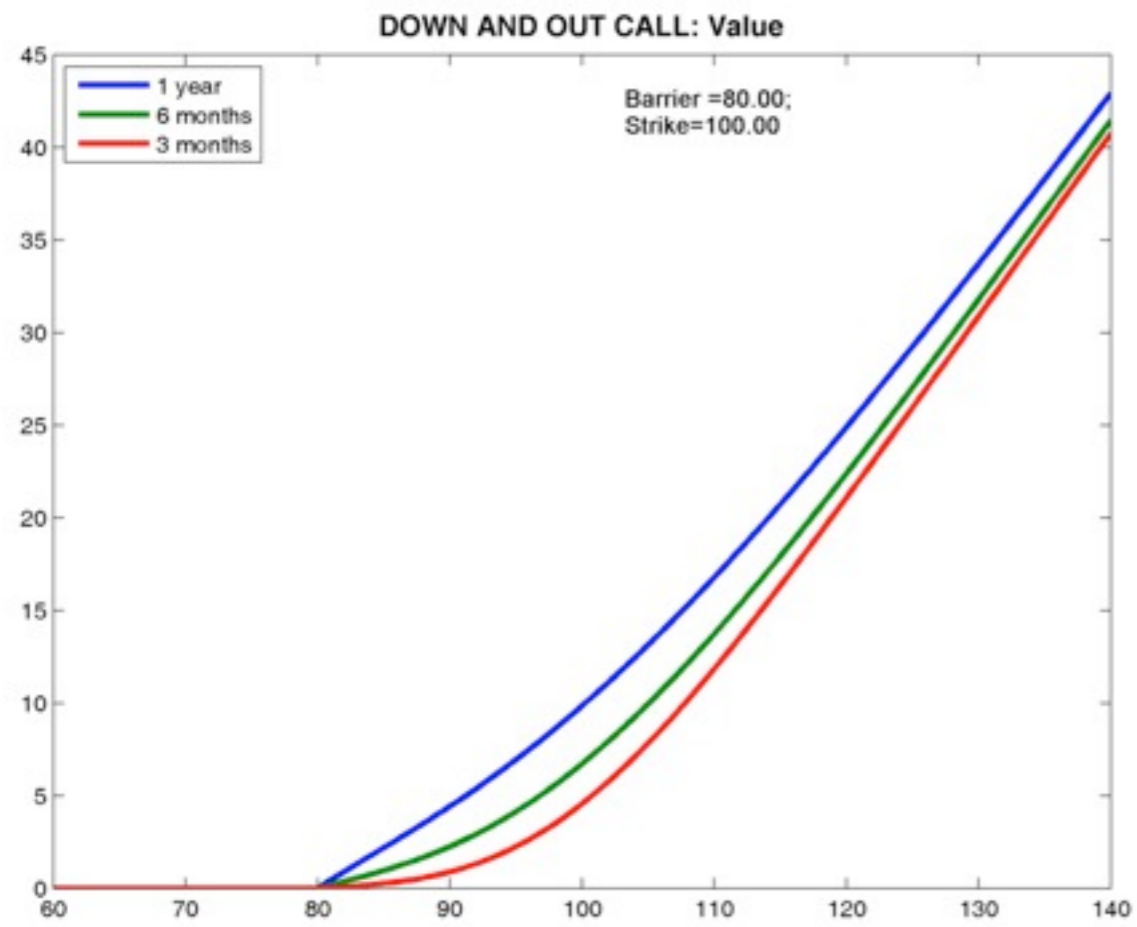
Just above barrier, delta is far less than that of vanilla put. The chance of knock-in is reduced when underlying price increases; leading to decline in option value.

- Gamma is large and positive near barrier and infinite at the barrier; vega also positive.



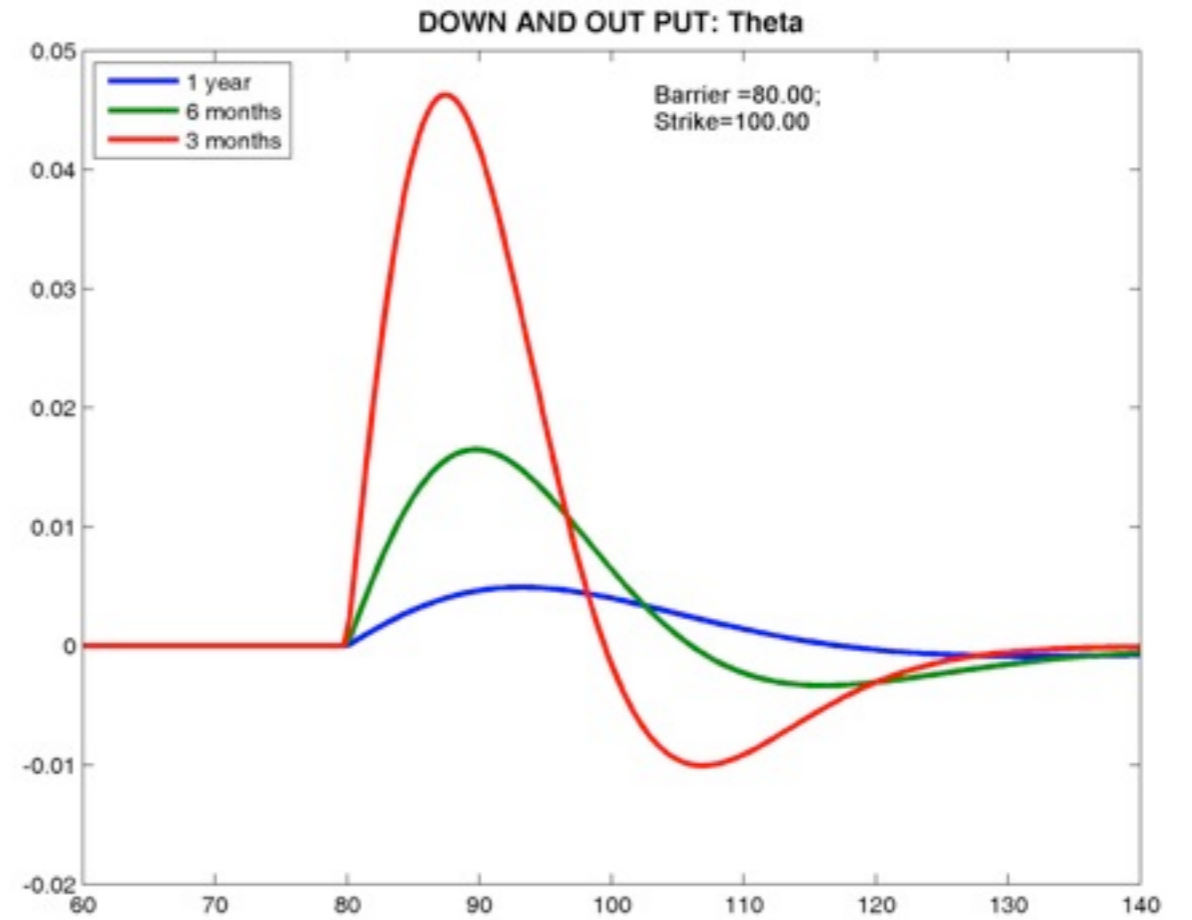
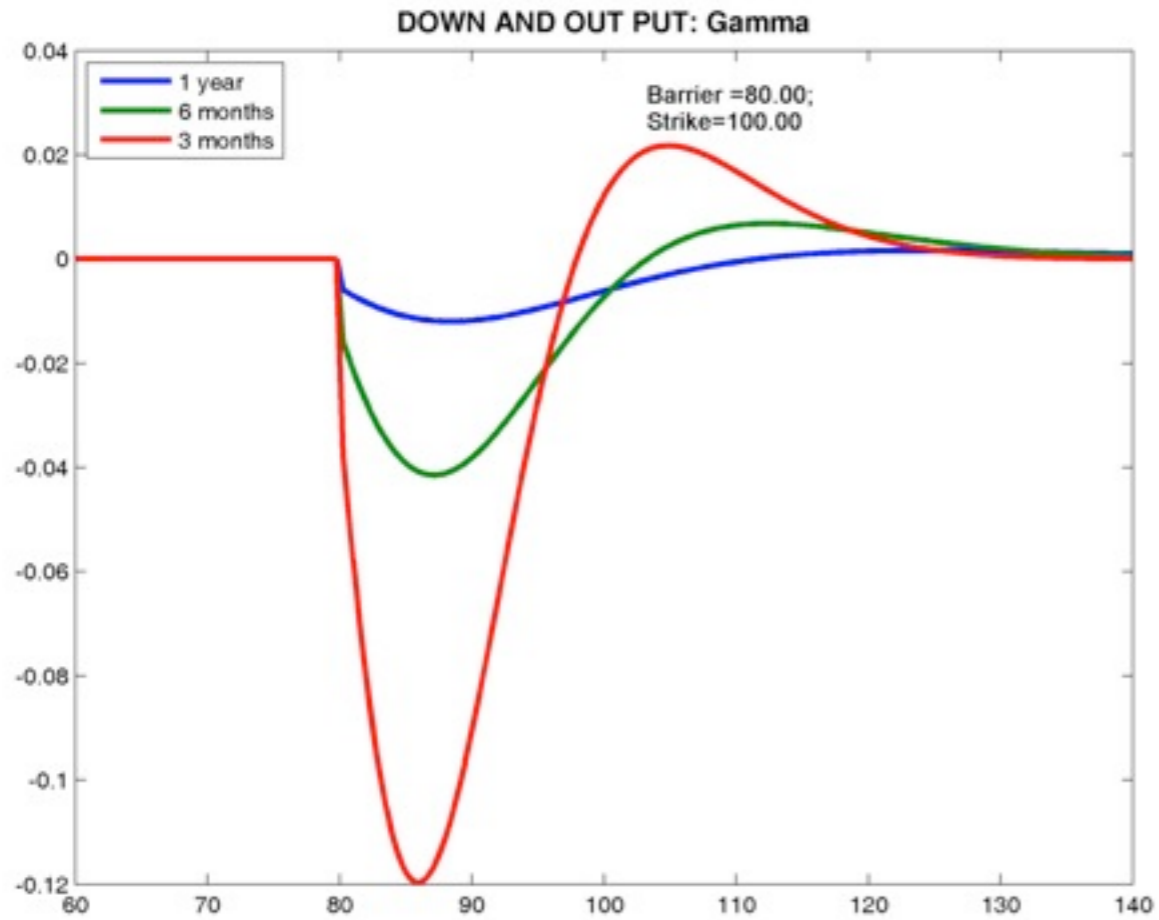
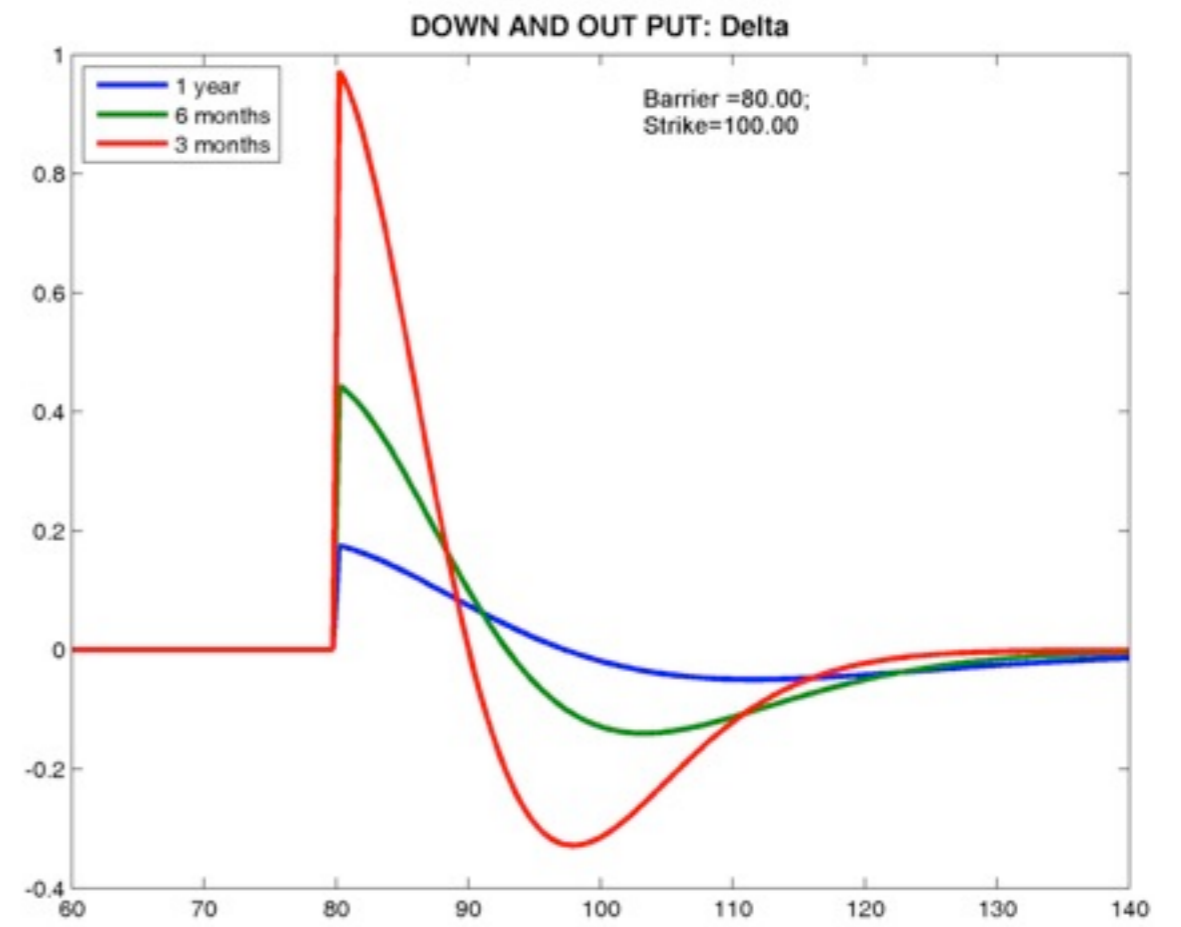
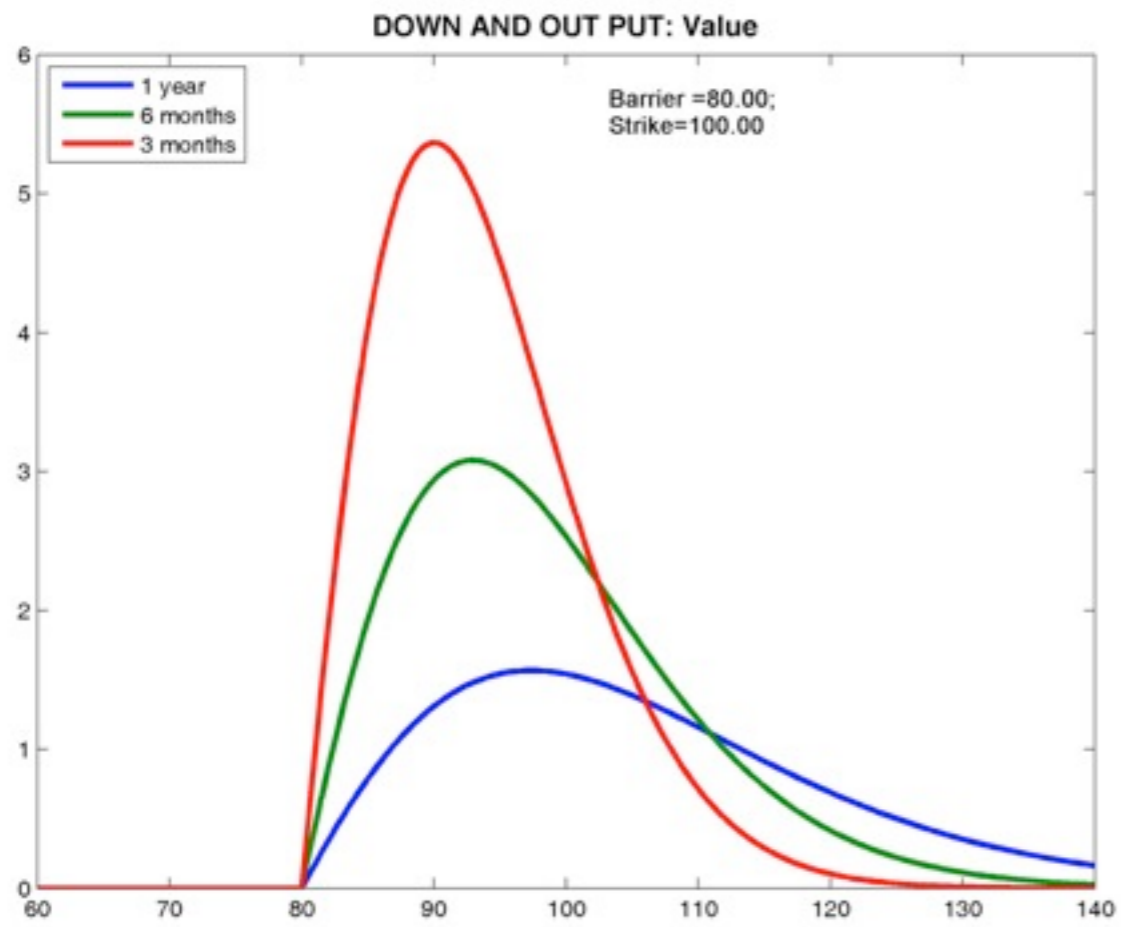
Down and Out Call

- Worth nearly the same as European call if barrier is far, because it is knocked out when call has left value left.
- Below the barrier delta is zero (call is worth 0); above the barrier call price goes up fast. Delta higher than that of vanilla call just above the barrier. Gamma is extremely large at the barrier. Option decreases as volatility increases!
- Delta decreases as stock price price increases just slightly higher than barrier.
- When the underlying is very far from barrier, the barrier has little effect and value, delta gamma all are like that of the vanilla call.



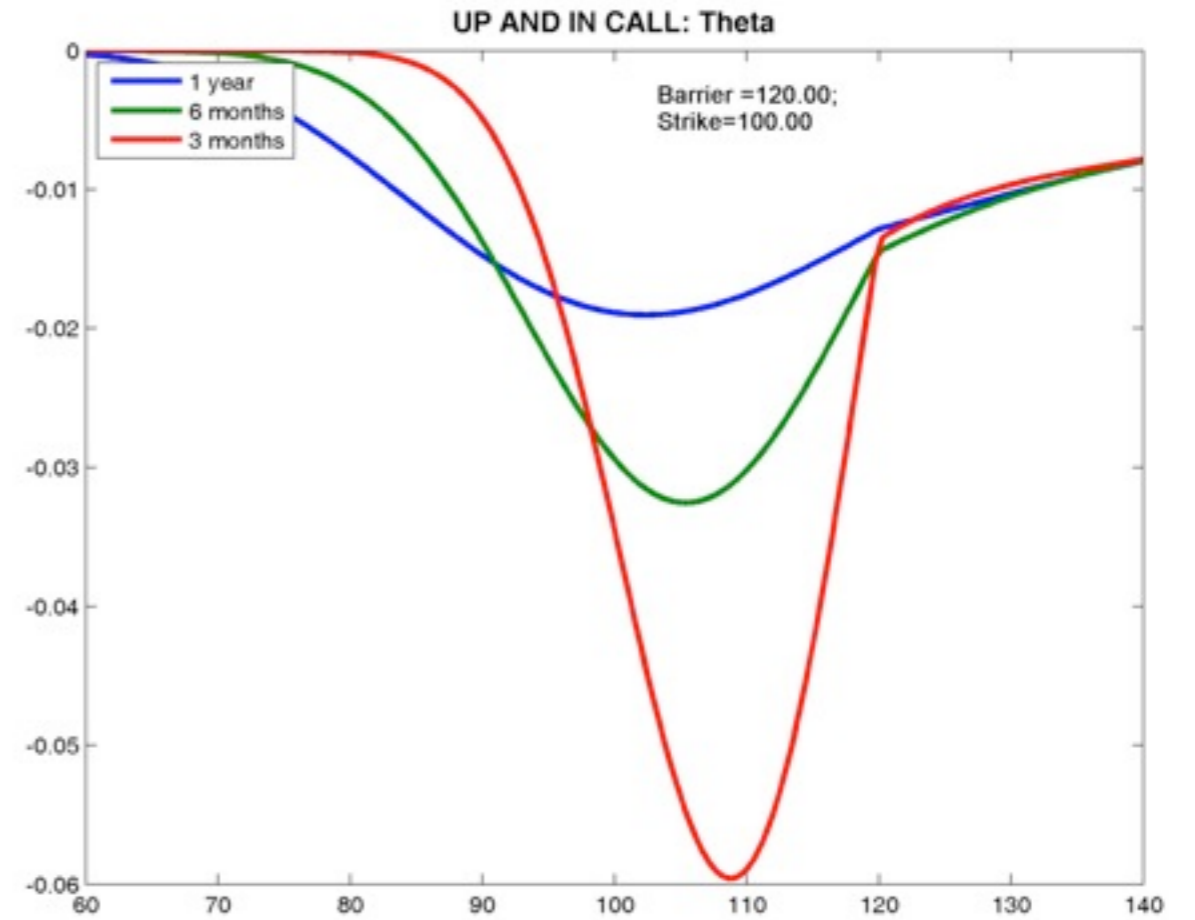
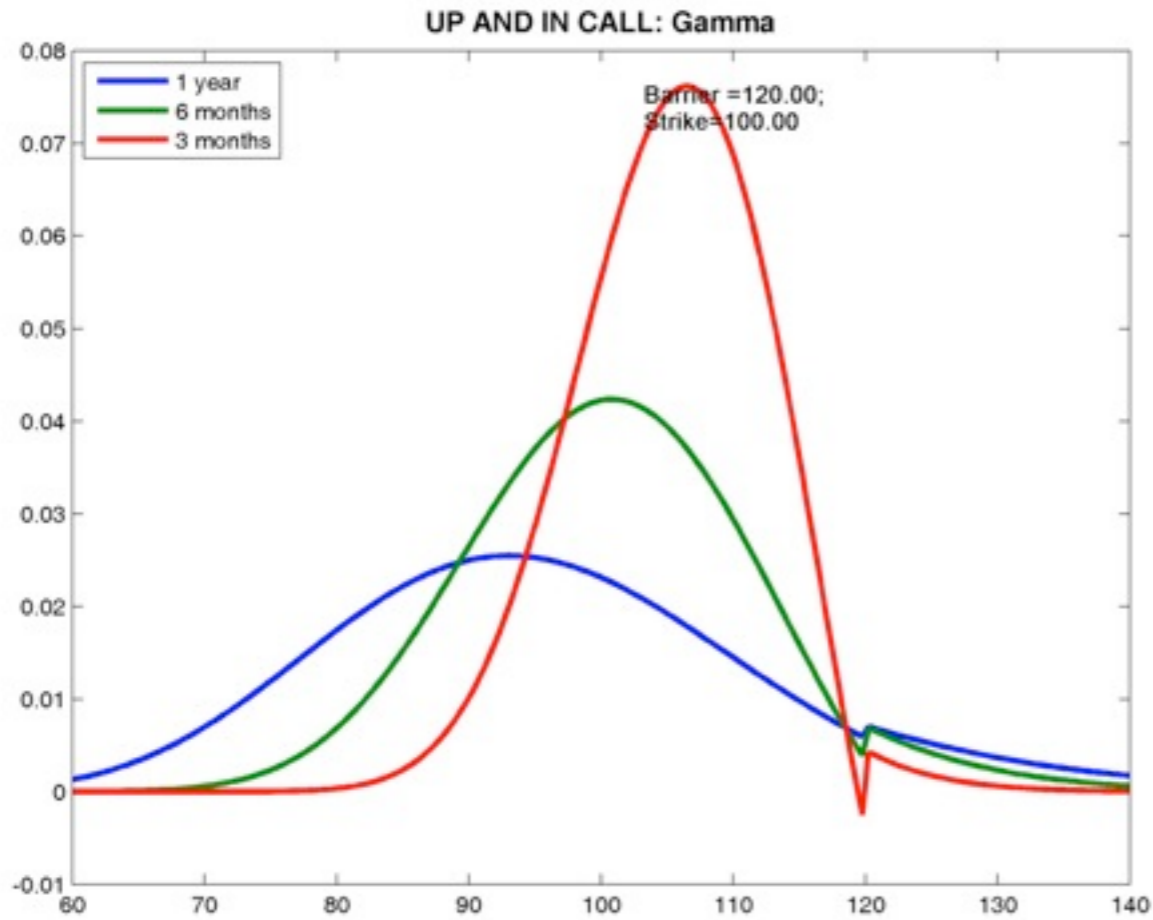
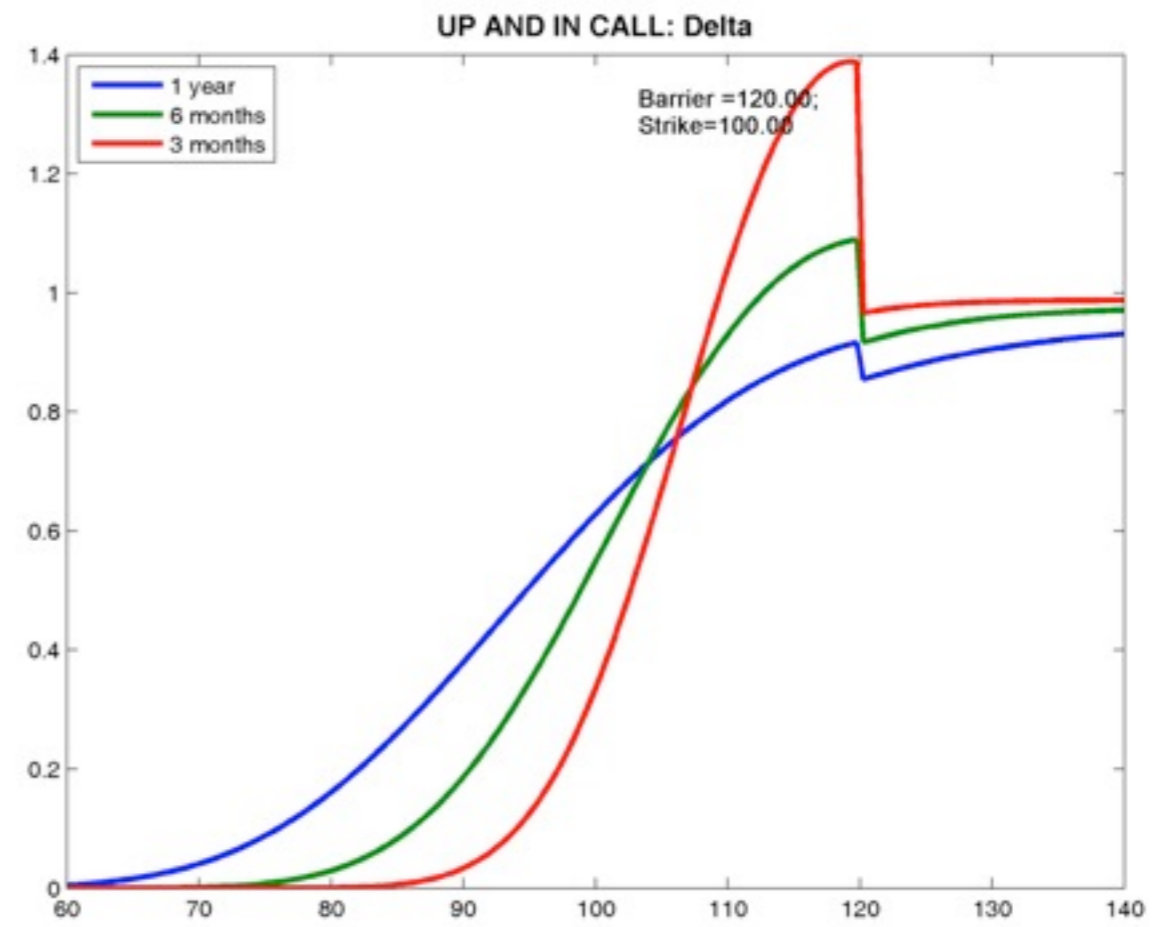
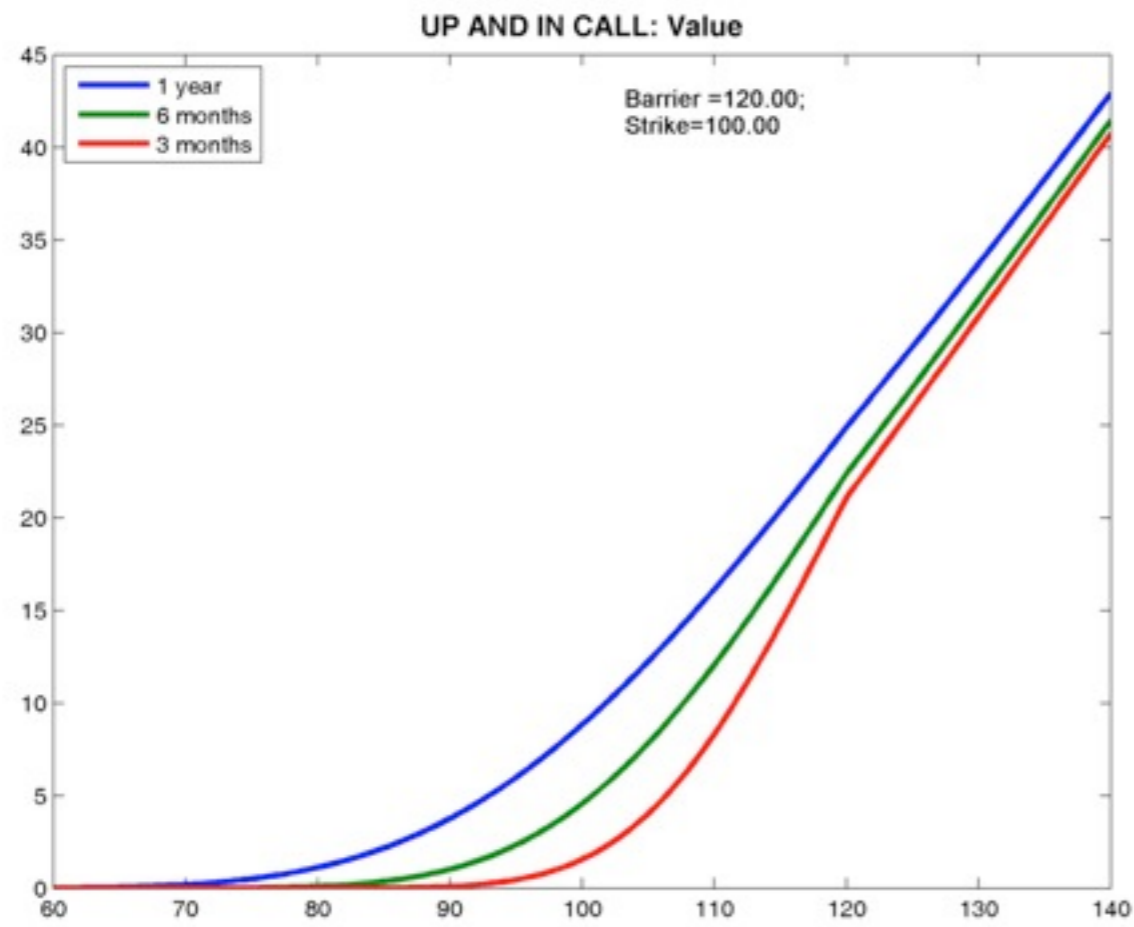
Down and Out Put

- The Down and Out put will get knocked out when the stock has moved down and the European put is in the money. Therefore it is priced much cheaper than standard put. If strike is below the barrier it is worthless.
- Just above the barrier, delta is large and positive- if price moves slightly higher the probability of knock-out is reduced. At much higher price levels, the barrier's impact is less and delta is negative like that of the vanilla put.
- Gamma is very large and negative at the barrier. Also short veg at the barrier.



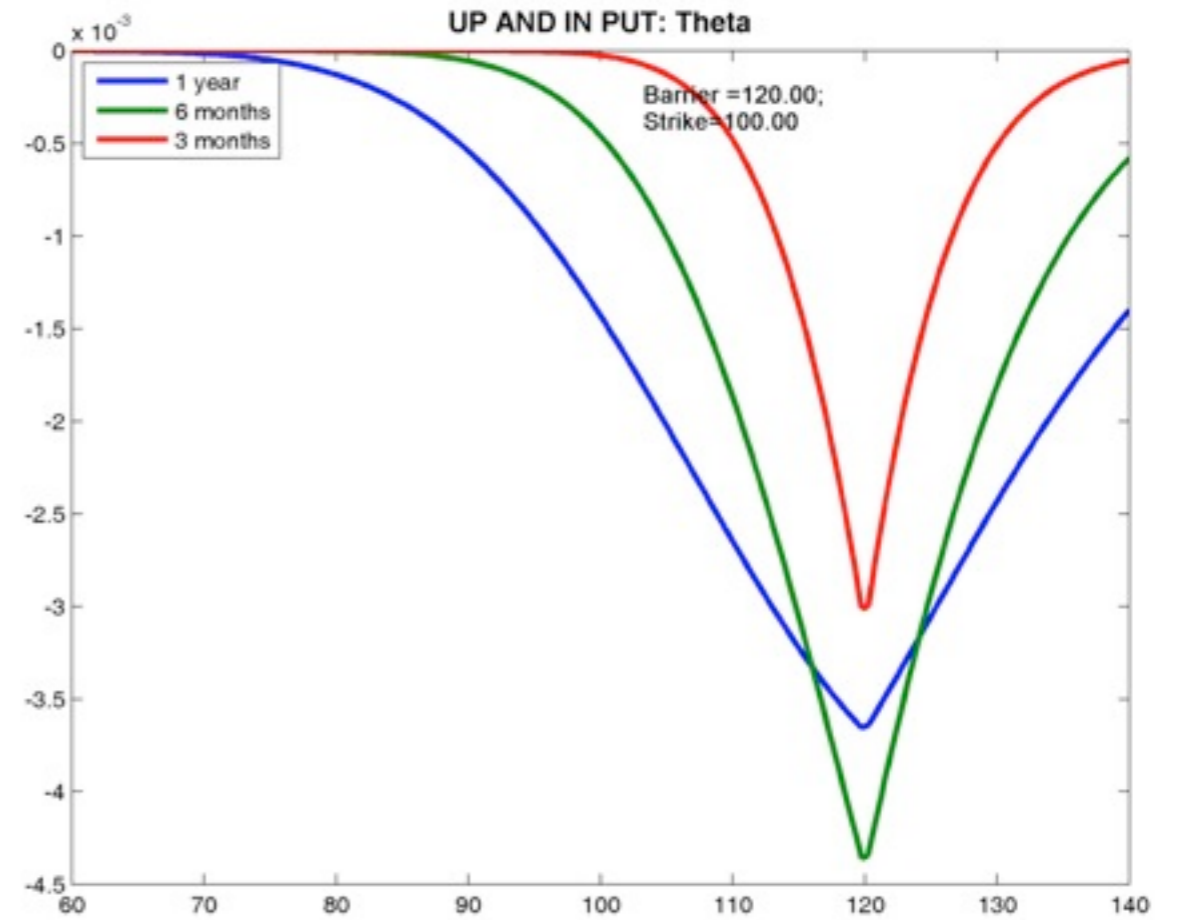
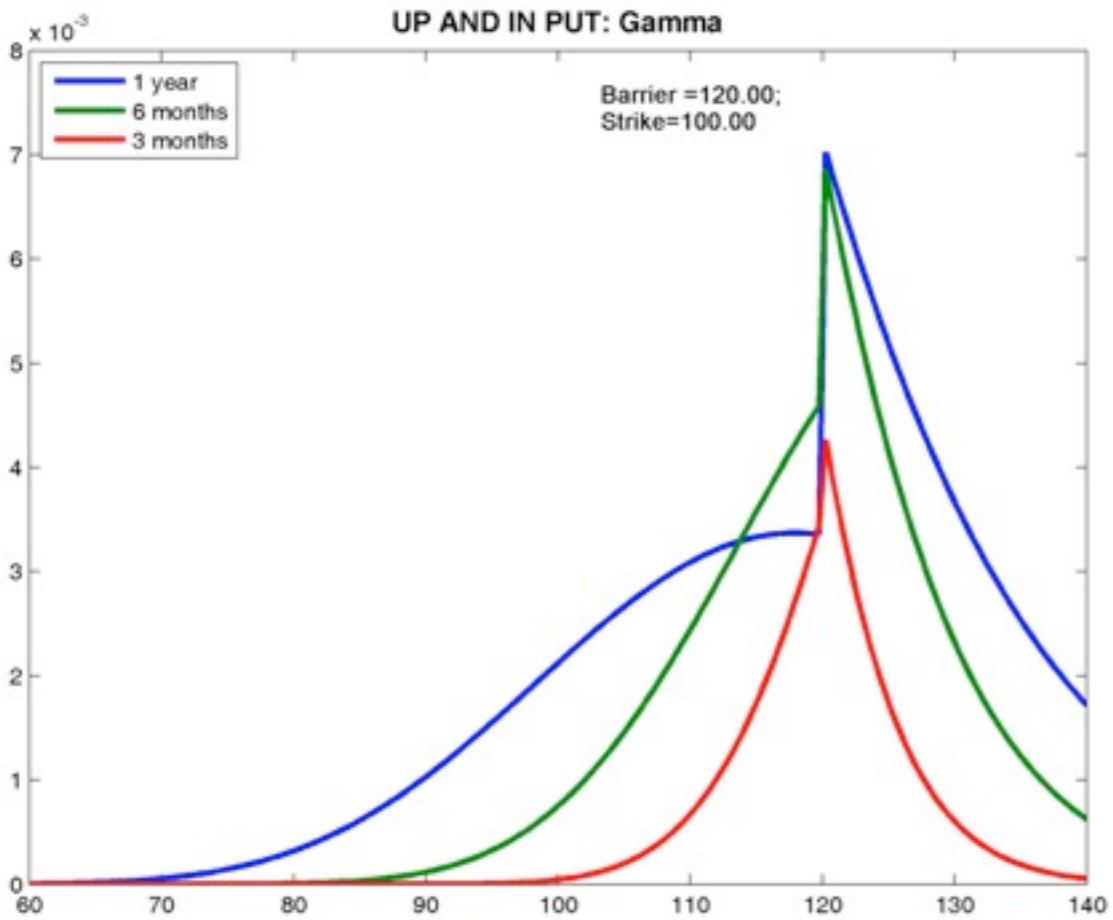
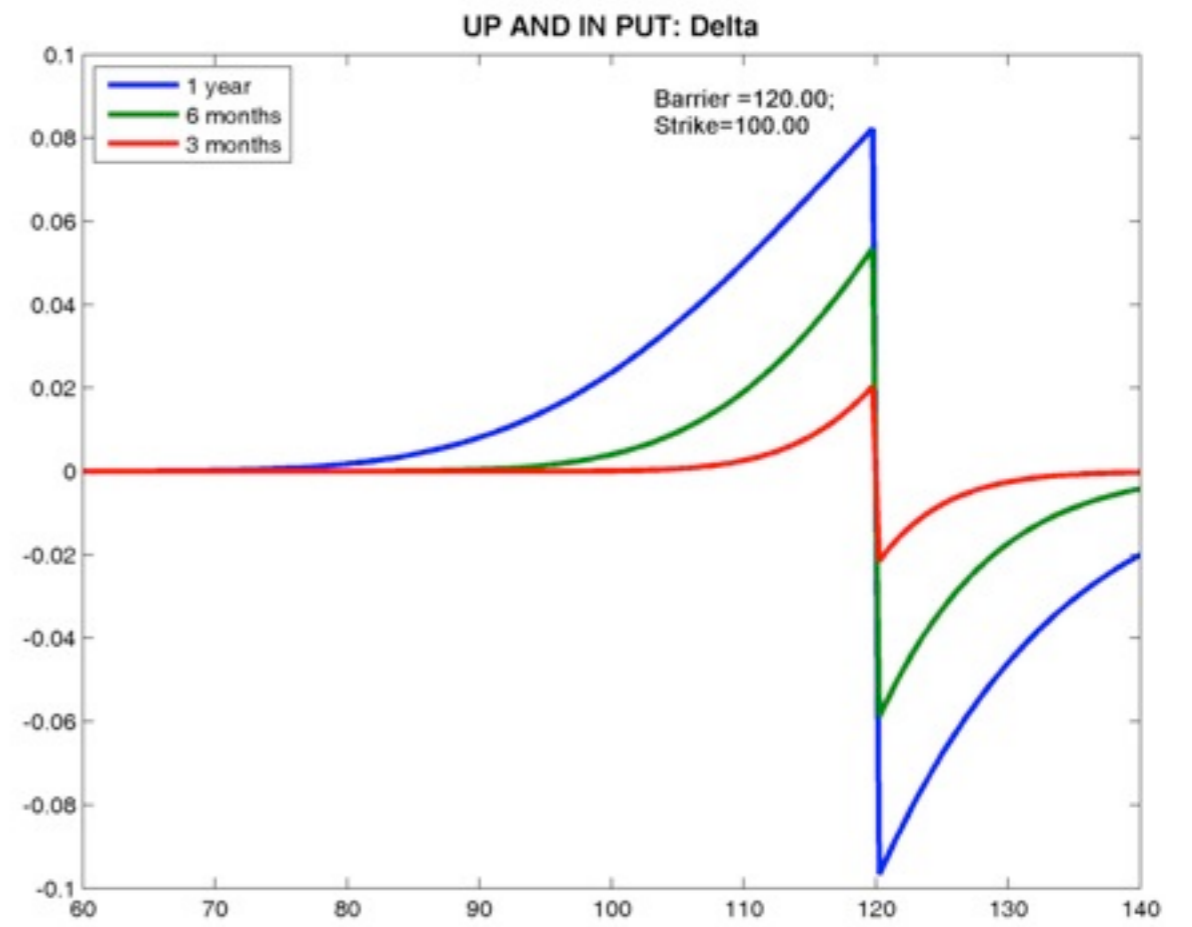
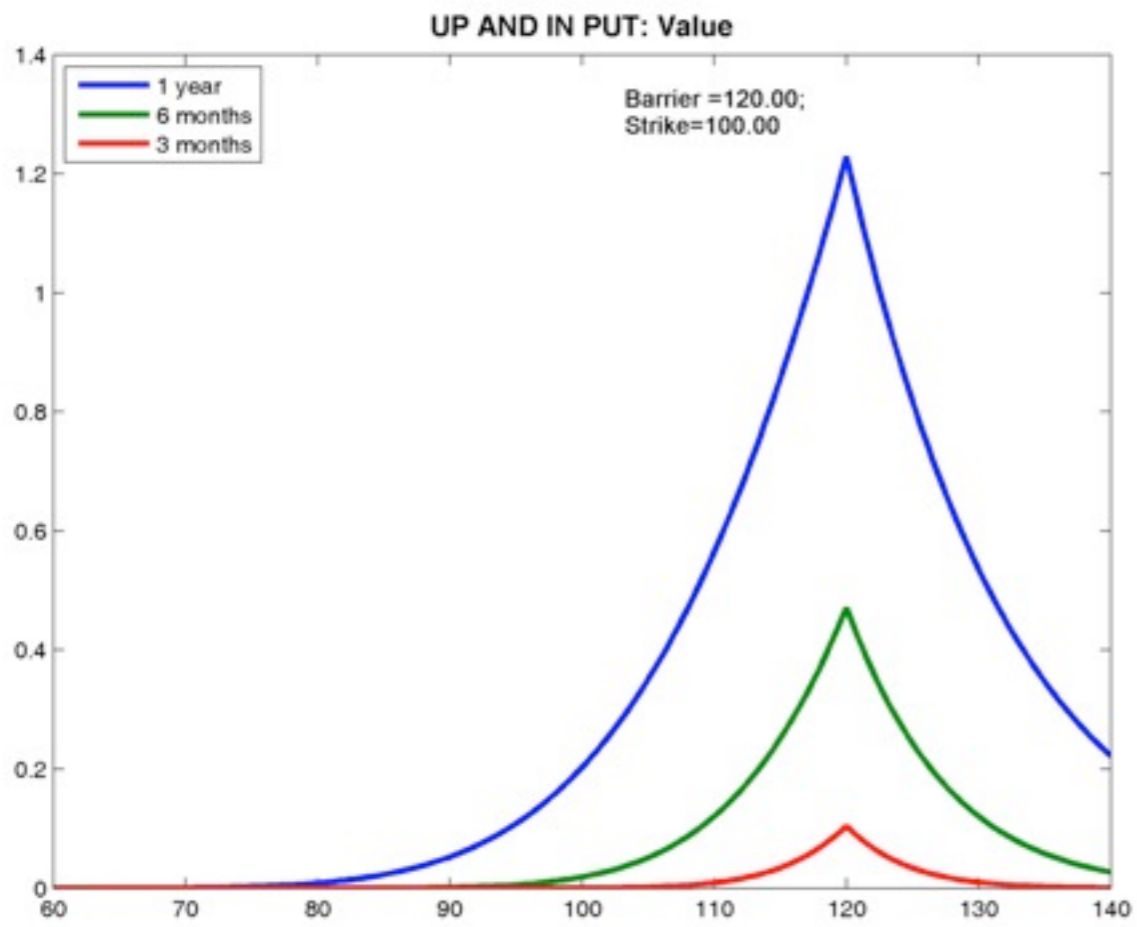
Up and In Call

- If strike is above the barrier, the Up and In Call is worth the same as Vanilla Call.
- It is not much cheaper than Vanilla call because it is knocked in for large moves that contribute to value of a vanilla call;
- As barrier is approached, it has large gain in value and delta is greater than of vanilla call, in some region even greater than 1.
- Gamma is very large for same reason



Up and In Put

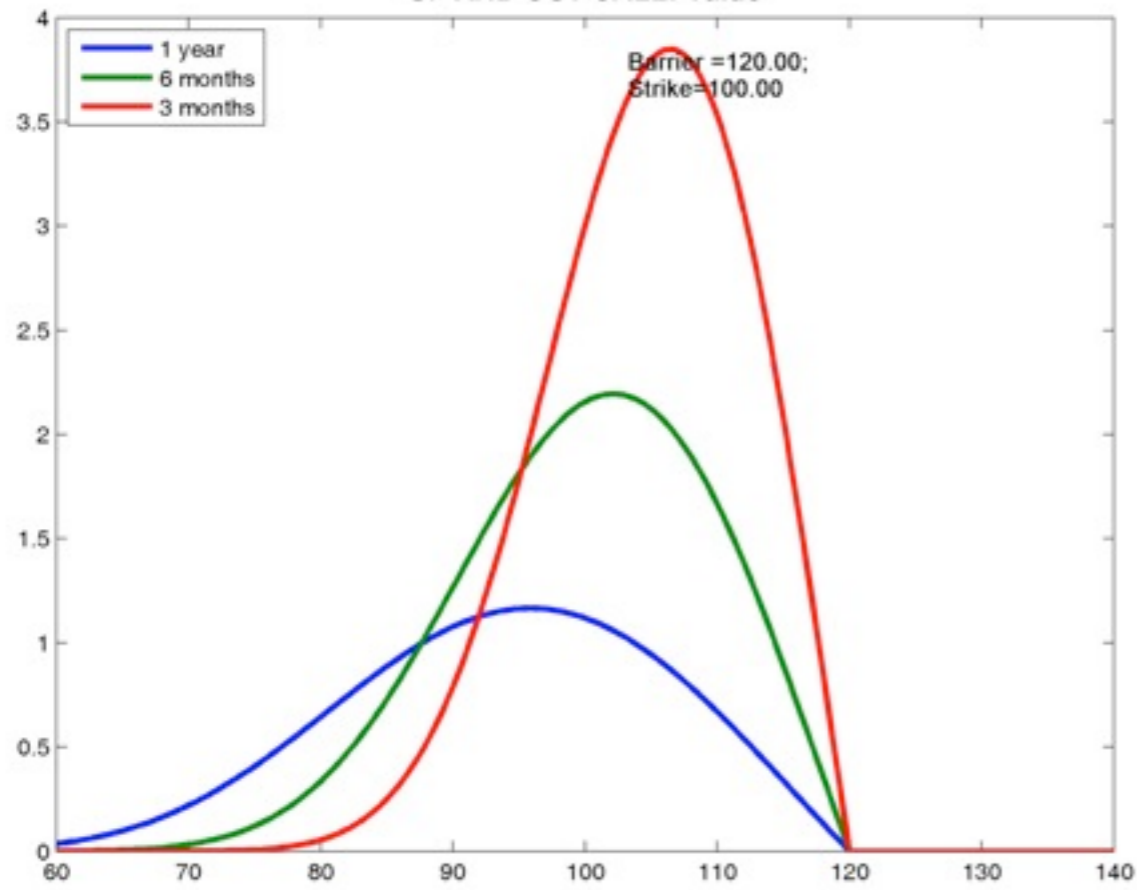
- Up and In Put is much cheaper than the standard put because it becomes alive only when the underlying has moved up and the standard put is worth very little.
- T the barrier, the option is equal in value to vanilla put, and after knock-in, it declines with rising stock price just as a vanilla put. The maximum value is at the barrier.
- Gamma is positive and extremely large at the barrier. Vega is also very large at the barrier.



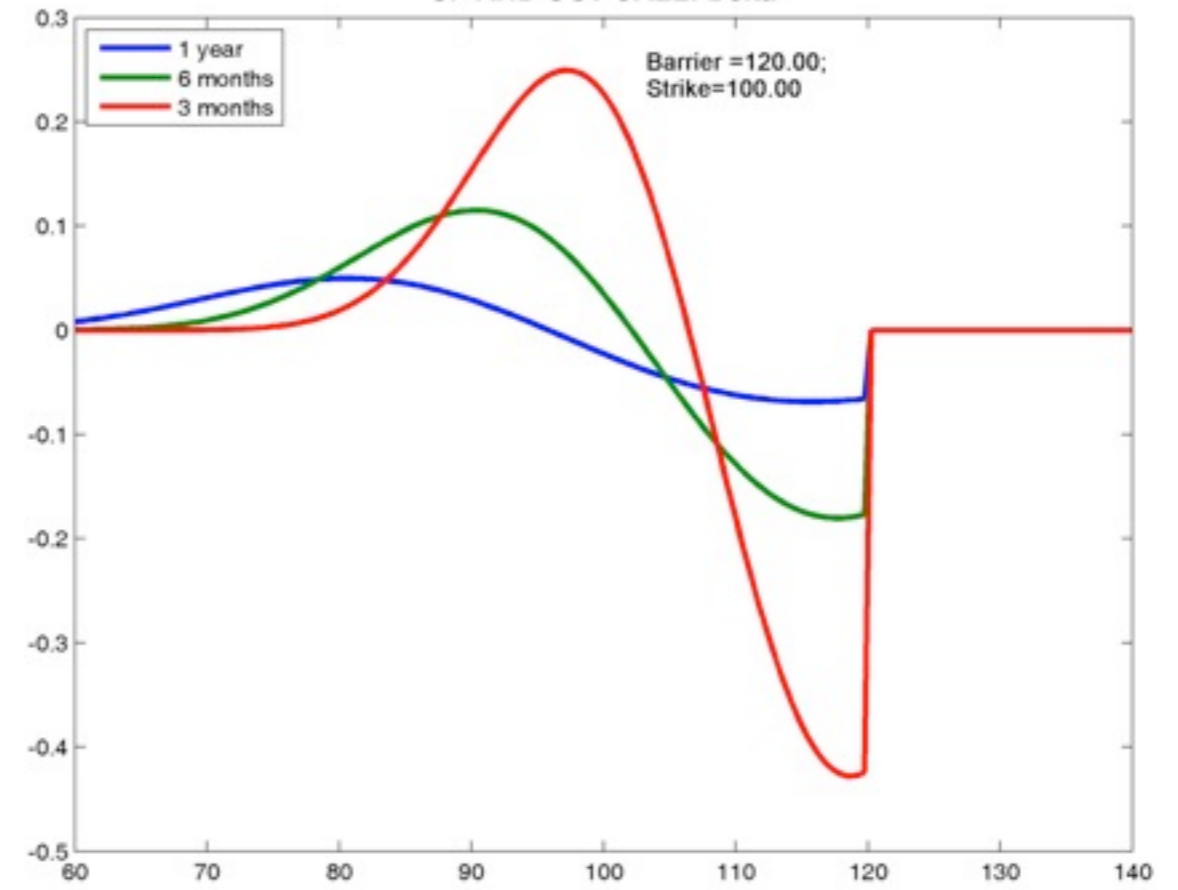
Up and Out Call

- The Up and Out Call is much cheaper than the Vanilla Call, because it is knocked out precisely when large up moves occur - and these are the moves which will contribute to the vanilla call's value. If strike is above the barrier, this option is worth zero.
- Much below barrier, delta is positive just like vanilla, but as barrier is approached, delta becomes negative; after barrier is crossed delta is zero.
- Gamma is infinite at barrier but we can only plot the gamma slightly to left and slightly to the right of barrier. If price of underlying is just below the barrier, delta hedging is a killer because for a small up move gamma goes to zero.

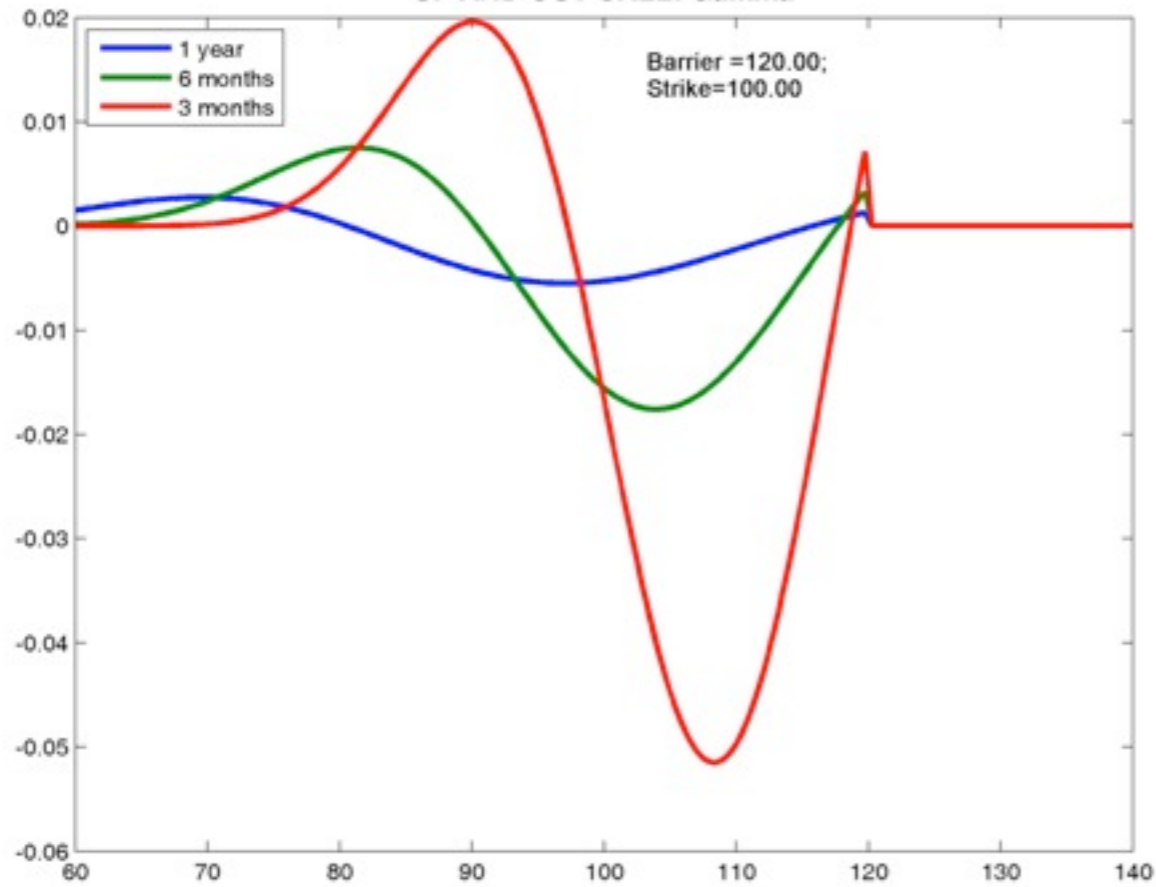
UP AND OUT CALL: Value



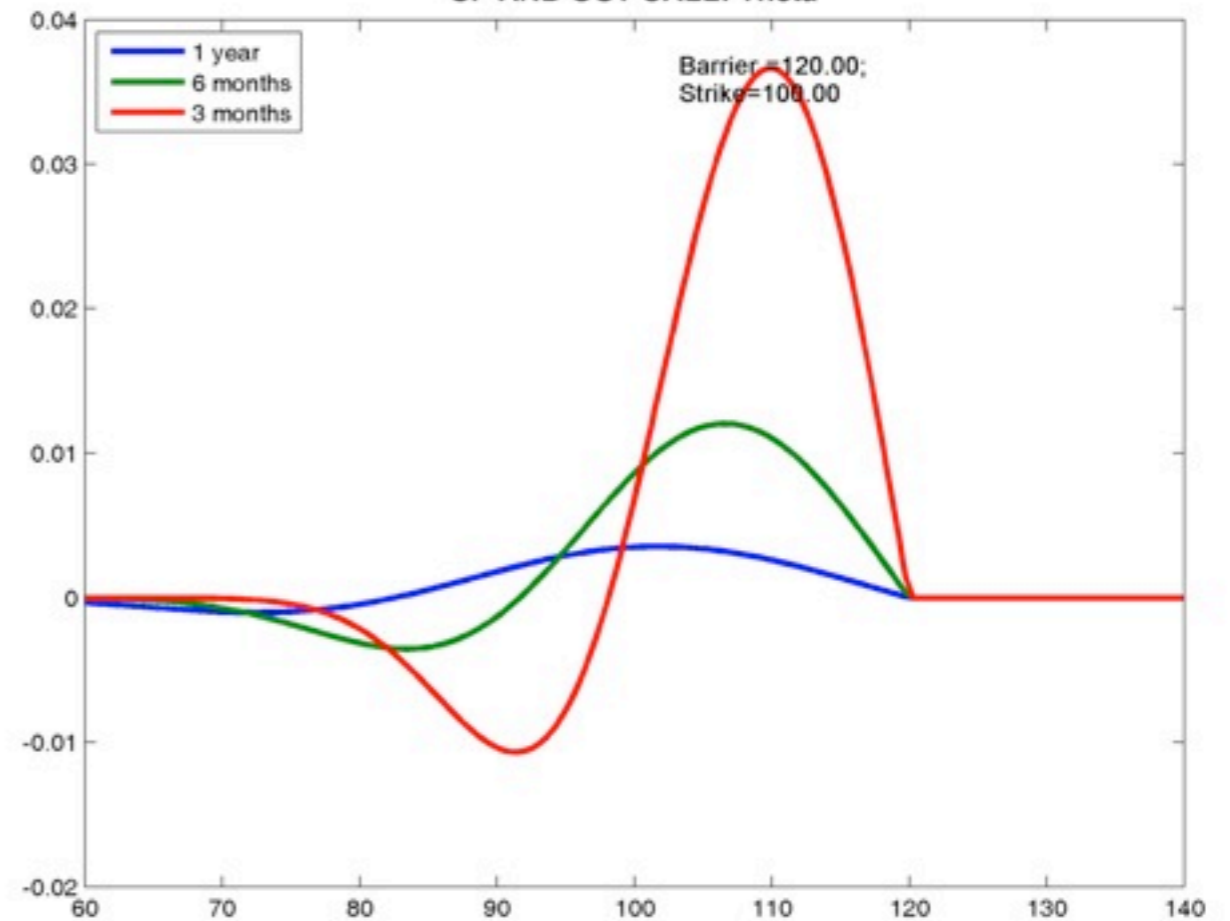
UP AND OUT CALL: Delta



UP AND OUT CALL: Gamma



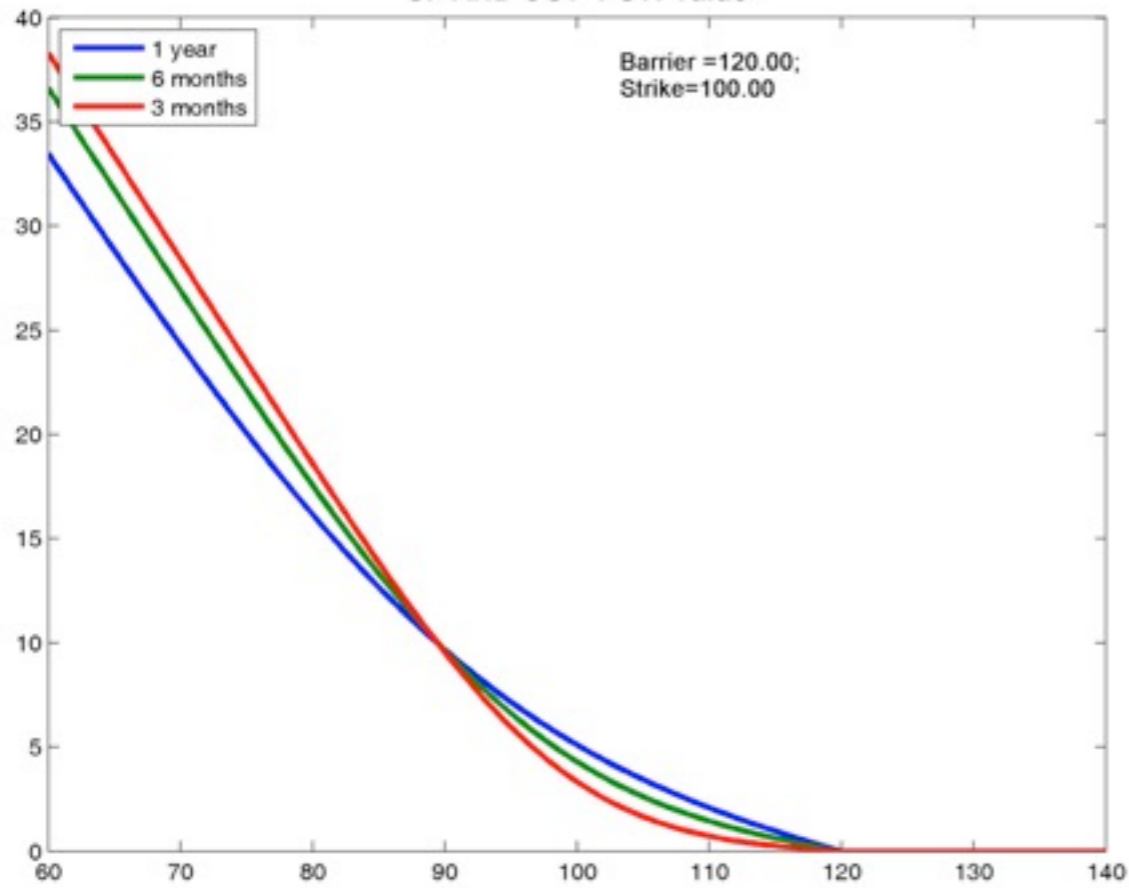
UP AND OUT CALL: Theta



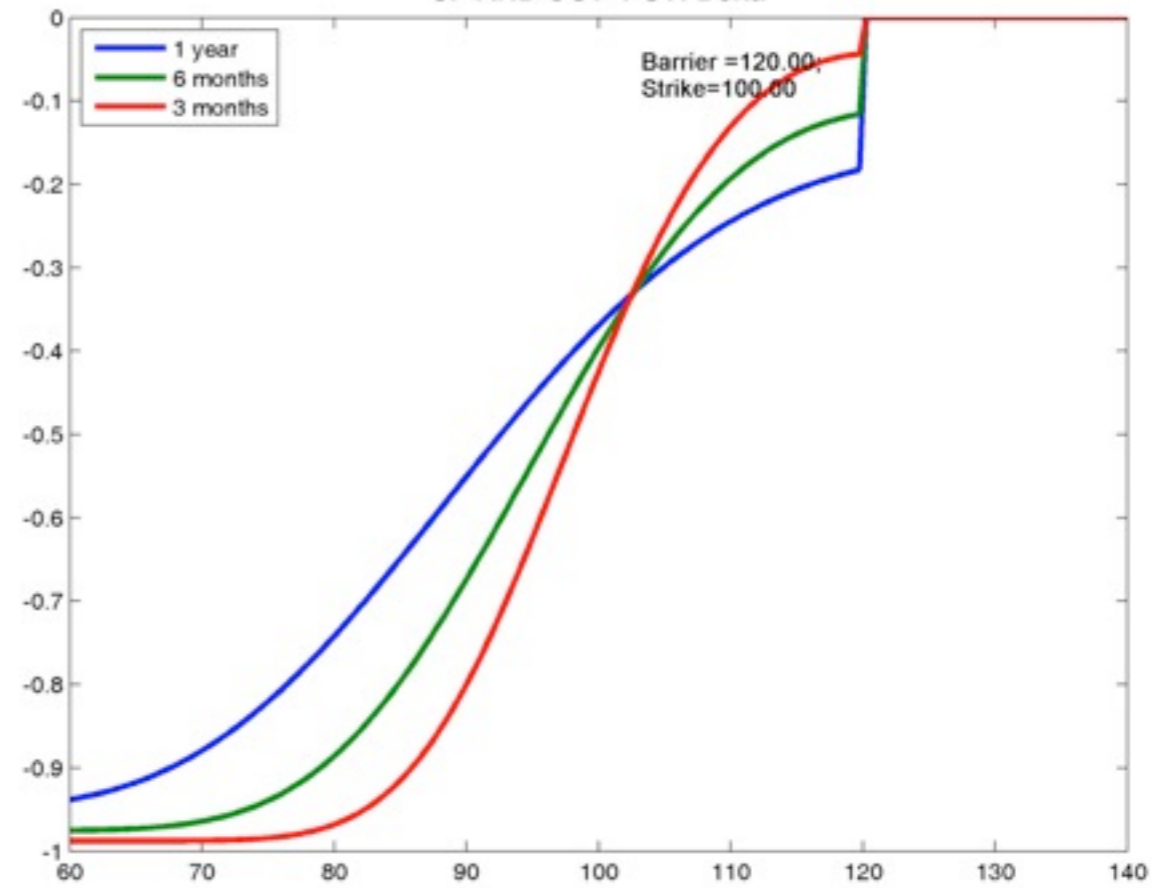
Up and Out Put

- Up and Out Put would be similar to Vanilla Put , because when it is knocked out owing to large up moves, the vanilla option might be worth little anyway.
- Below the barrier, delta is similar to vanilla put but above the barrier it is zero as option is knocked out.
- Likewise, gamma is similar to that of vanilla put below the barrier and zero above - and it is infinite at the barrier

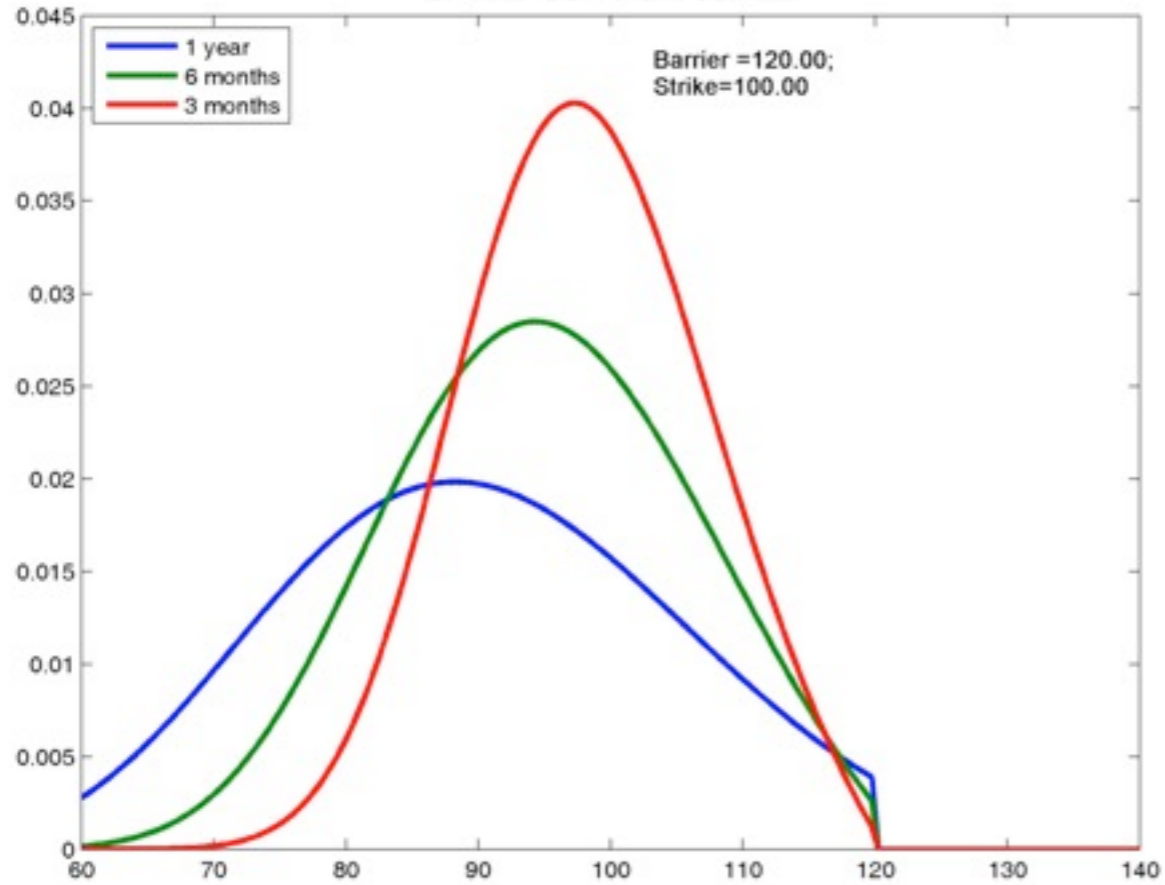
UP AND OUT PUT: Value



UP AND OUT PUT: Delta



UP AND OUT PUT: Gamma



UP AND OUT PUT: Theta

