Option Greeks:
A Detailed Graphical Treatment

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QF 301
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Greeks

- A Greek RISK MEASURE should not be thought of as a single number
- Rather, a range of numbers to be examined in different buckets and
- Under different scenarios

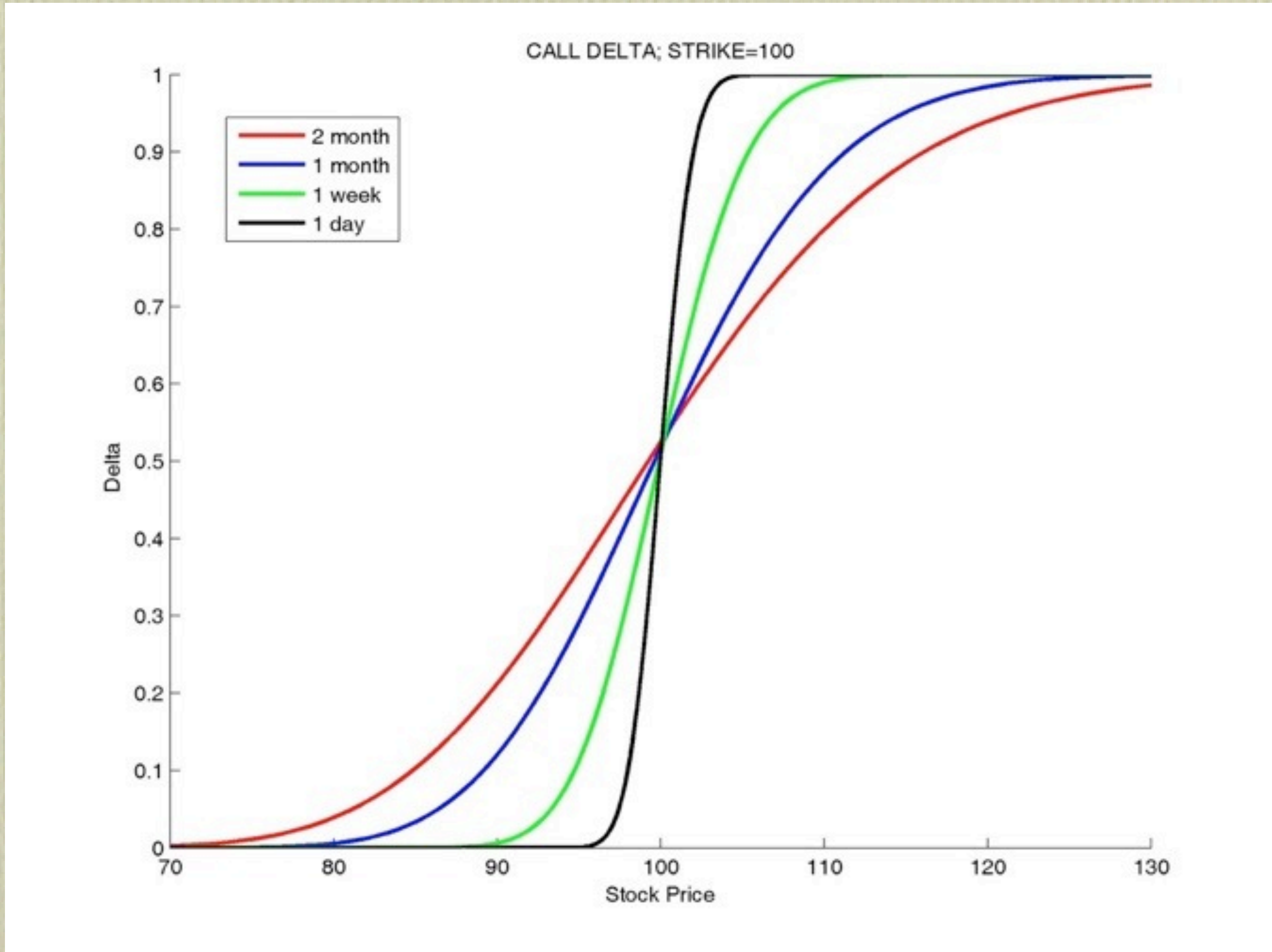
Delta

- Delta is the first partial derivative of an option with respect to the price of the underlying.
- Usually the first thing we want to control.

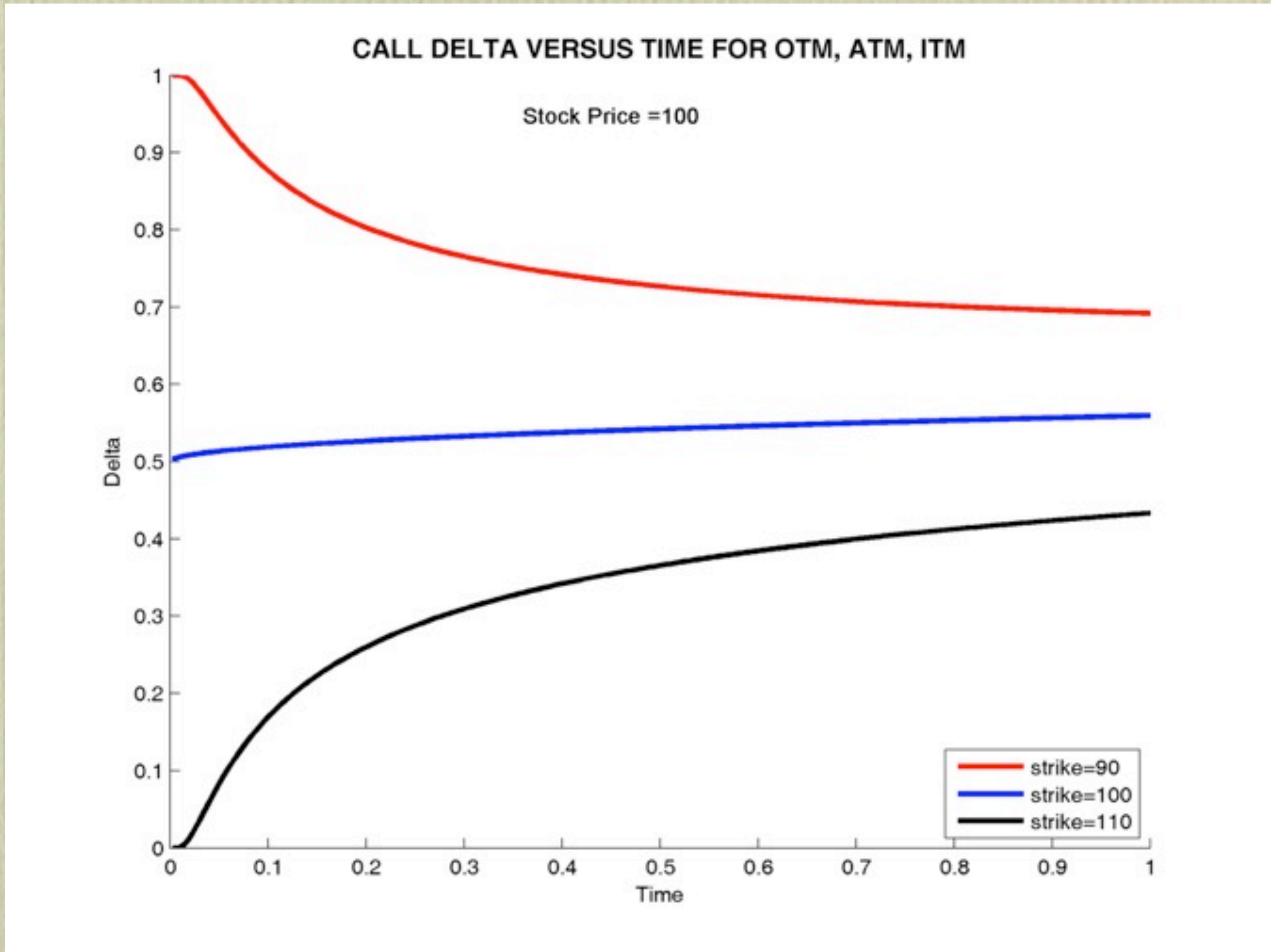
$$\Delta = \frac{\partial C}{\partial S}$$

Call Delta	$e^{-q\tau}\Phi(d_1)$
Put Delta	$-e^{-q\tau}\Phi(-d_1)$

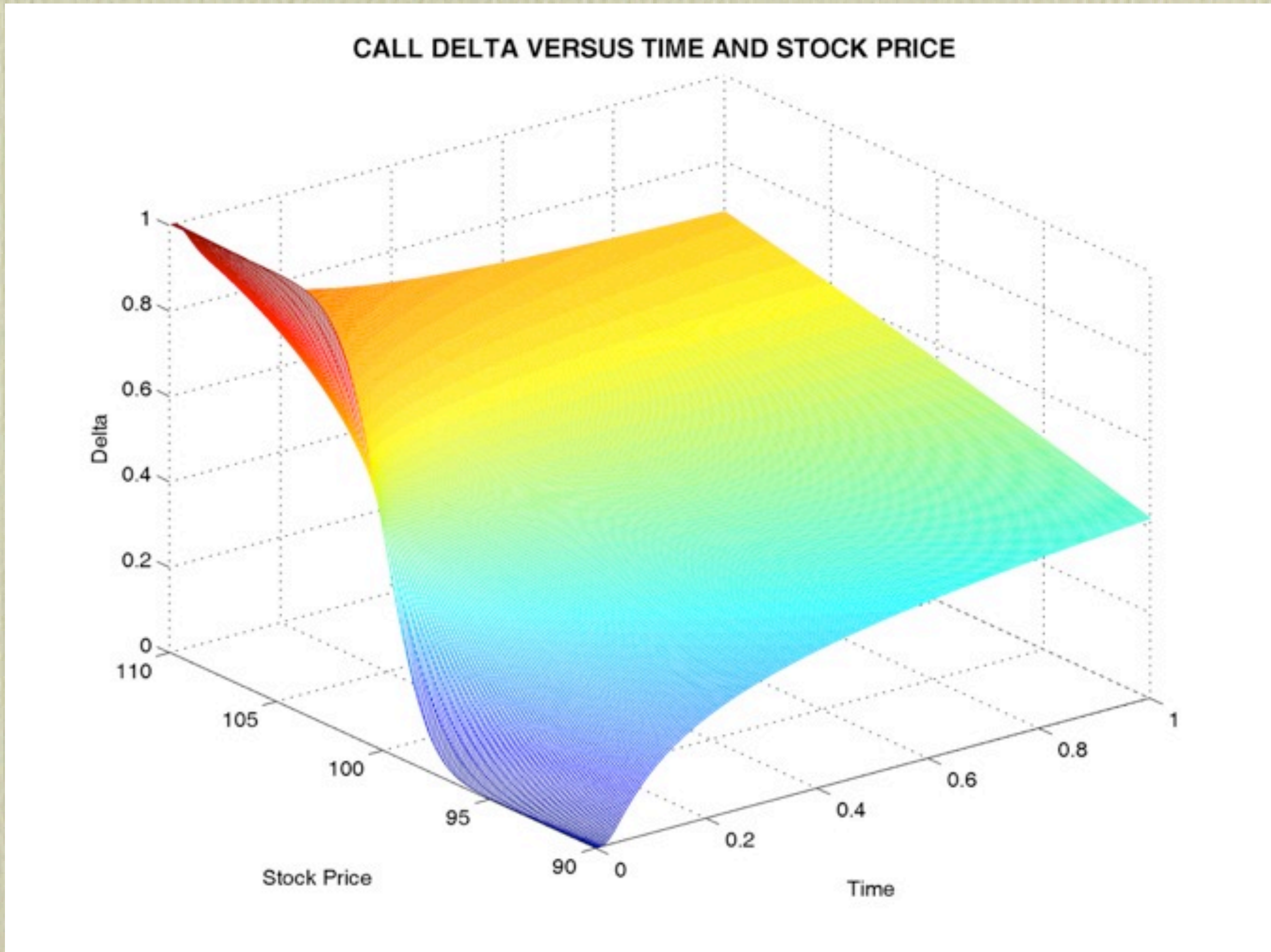
Delta of a Call Option



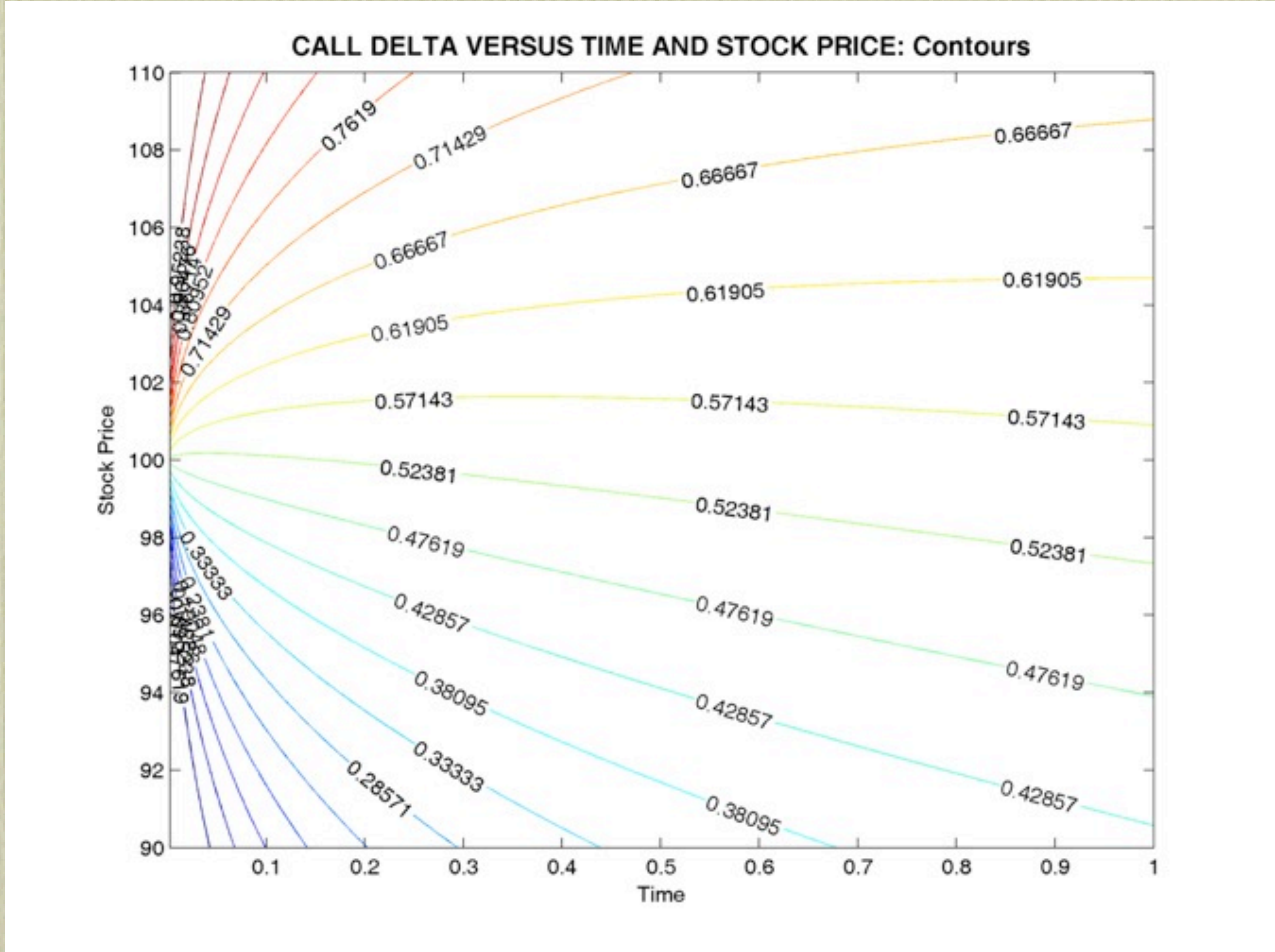
Delta vs Time for a CALL



Surface Plot for Delta vs S & T



Contour Plot for Delta vs S & T



Gamma

- Gamma is the second partial derivative of an option with respect to the price of the underlying.
- It is the first partial derivative of the option with respect to the underlying price.

$$\gamma = \frac{\partial^2 C}{\partial S^2}$$

Gamma (contd.)

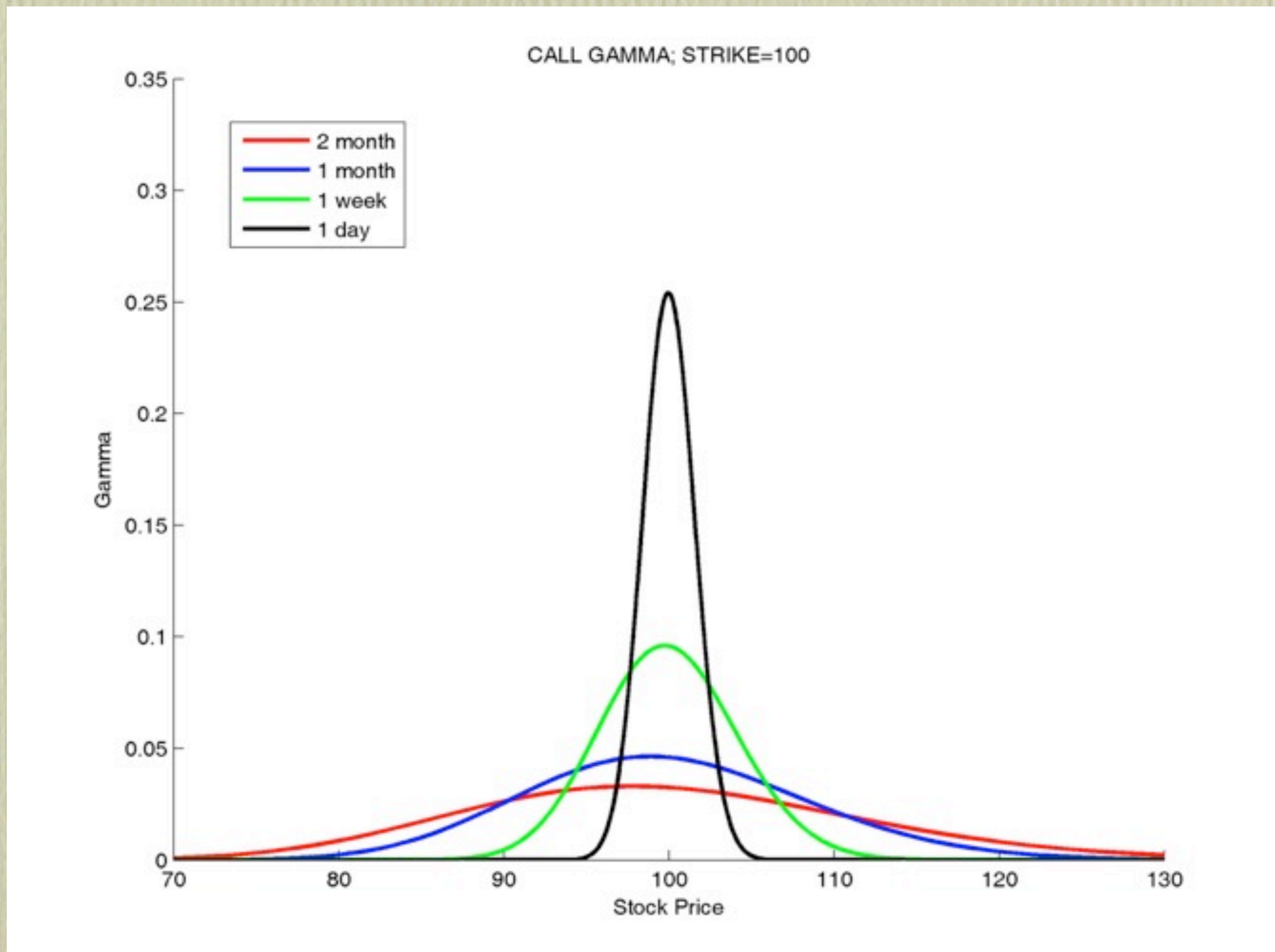
- Taking the derivative of delta with S, we get

$$\gamma = \frac{N'(d_1)}{S\sigma\sqrt{t}}$$

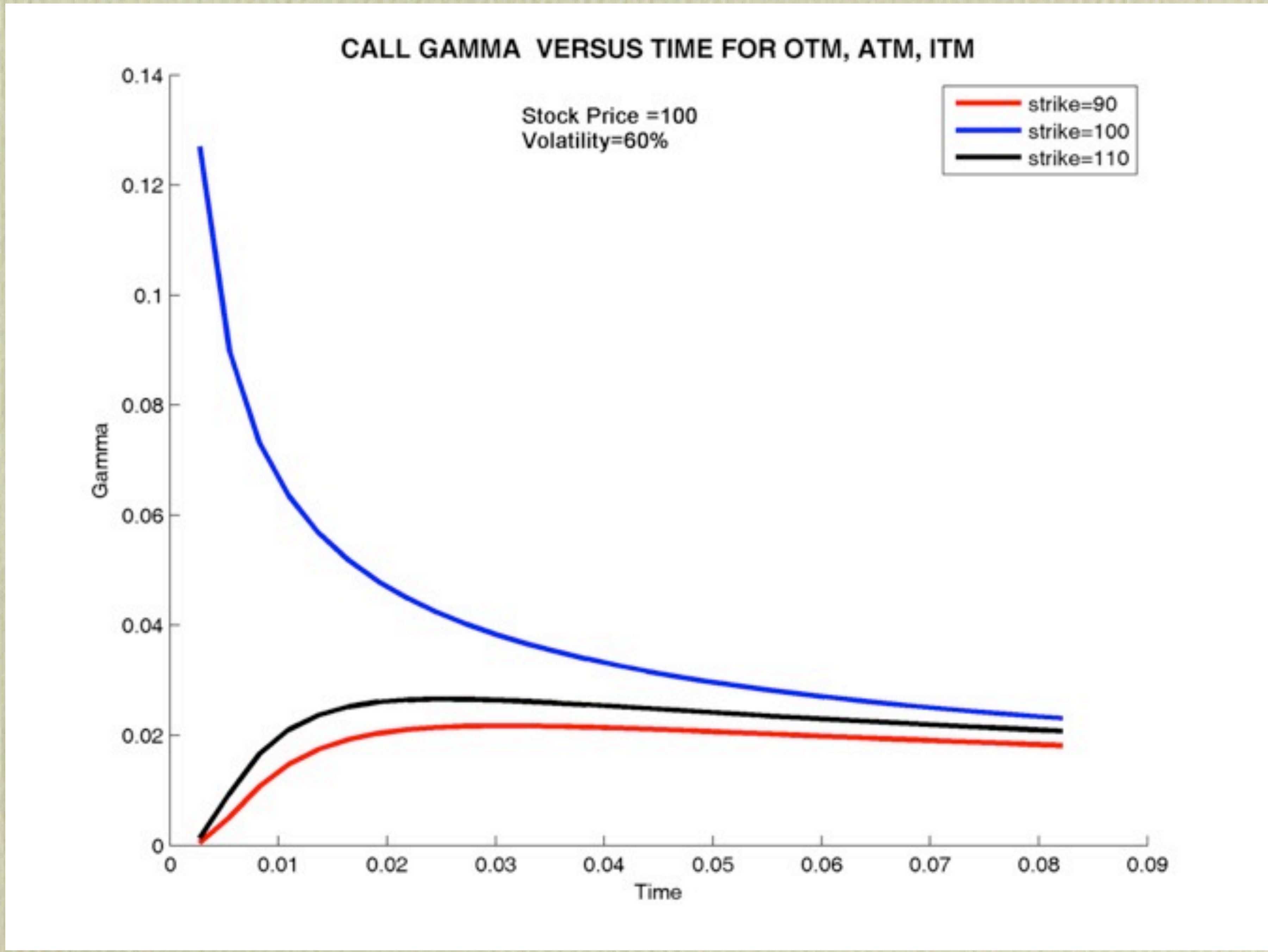
- Note that the Gamma of a European call and Put are the same.

- Often, we want to make a portfolio delta and gamma neutral. In this case, we first neutralize for gamma and then delta. For making delta, gamma and vega neutral, solve a system of linear equations. At least two options needed.

Variation of Gamma with Stock Price



Gamma vs Time for CALL



Theta

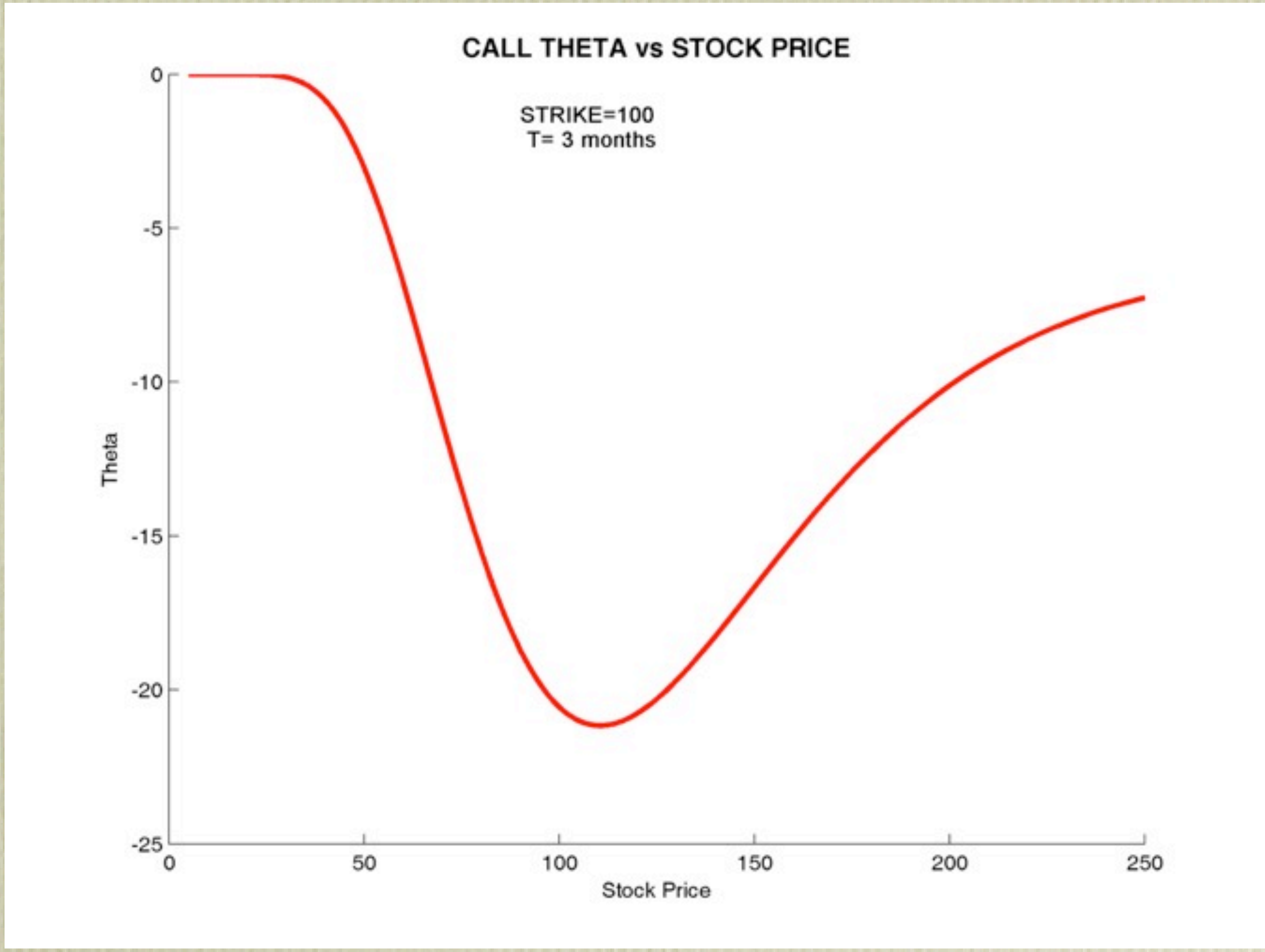
- Rate of change of option price w.r.t. time.

- Expression $$\theta = \frac{\partial C}{\partial t}$$

- Call Theta: $$-e^{-q\tau}\frac{S\varphi(d_1)\sigma}{2\sqrt{\tau}} - rKe^{-r\tau}\Phi(d_2) + qSe^{-q\tau}\Phi(d_1)$$

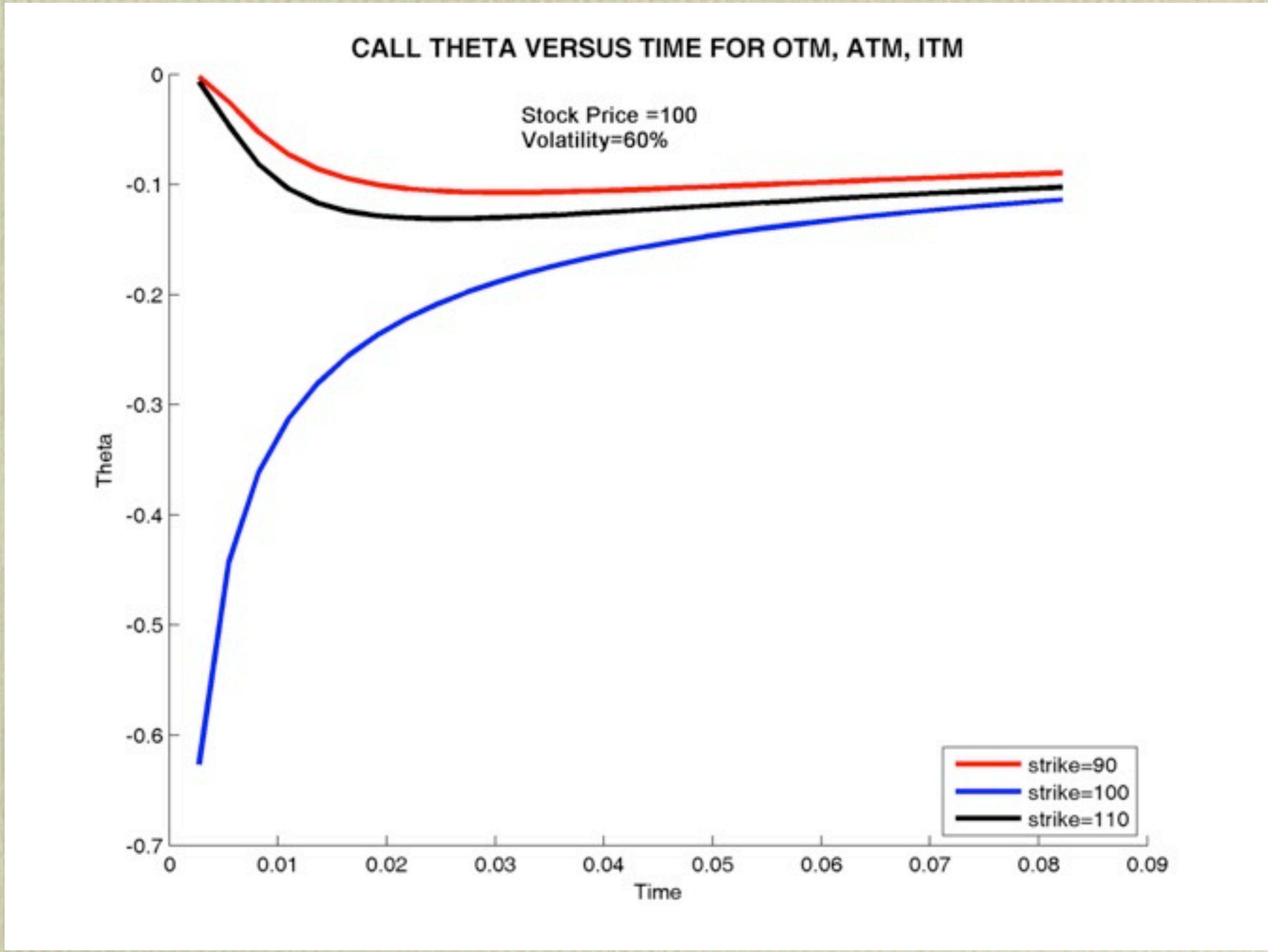
- Put Theta: $$-e^{-q\tau}\frac{S\varphi(d_1)\sigma}{2\sqrt{\tau}} + rKe^{-r\tau}\Phi(-d_2) - qSe^{-q\tau}\Phi(-d_1)$$

Theta



CALL THETA vs STOCK PRICE
STRIKE=100
T= 3 months
Theta
0
-5
-10
-15
-20
-25
0
50
100
150
200
250
Stock Price

Theta vs Time for CALLS

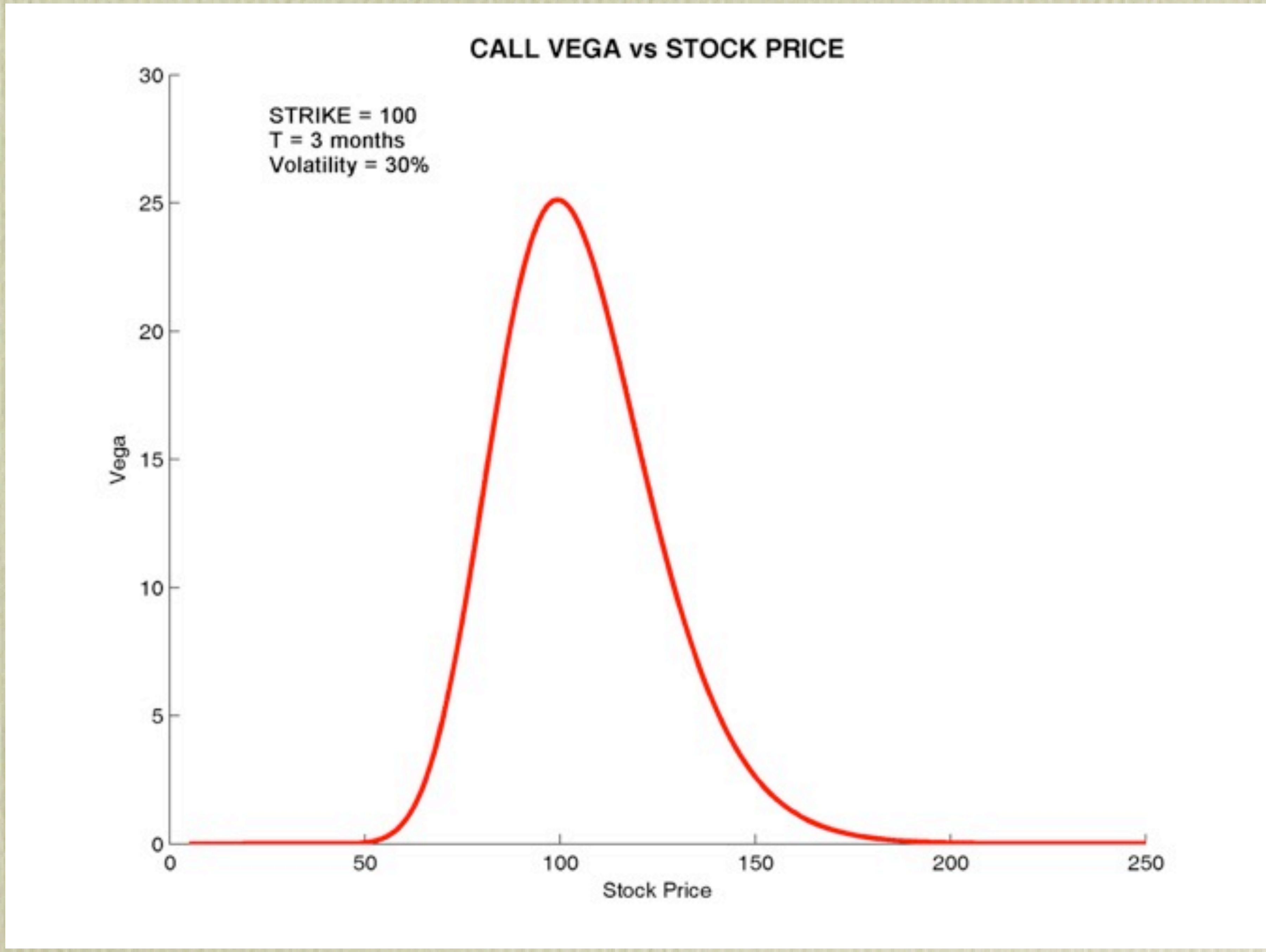


Vega

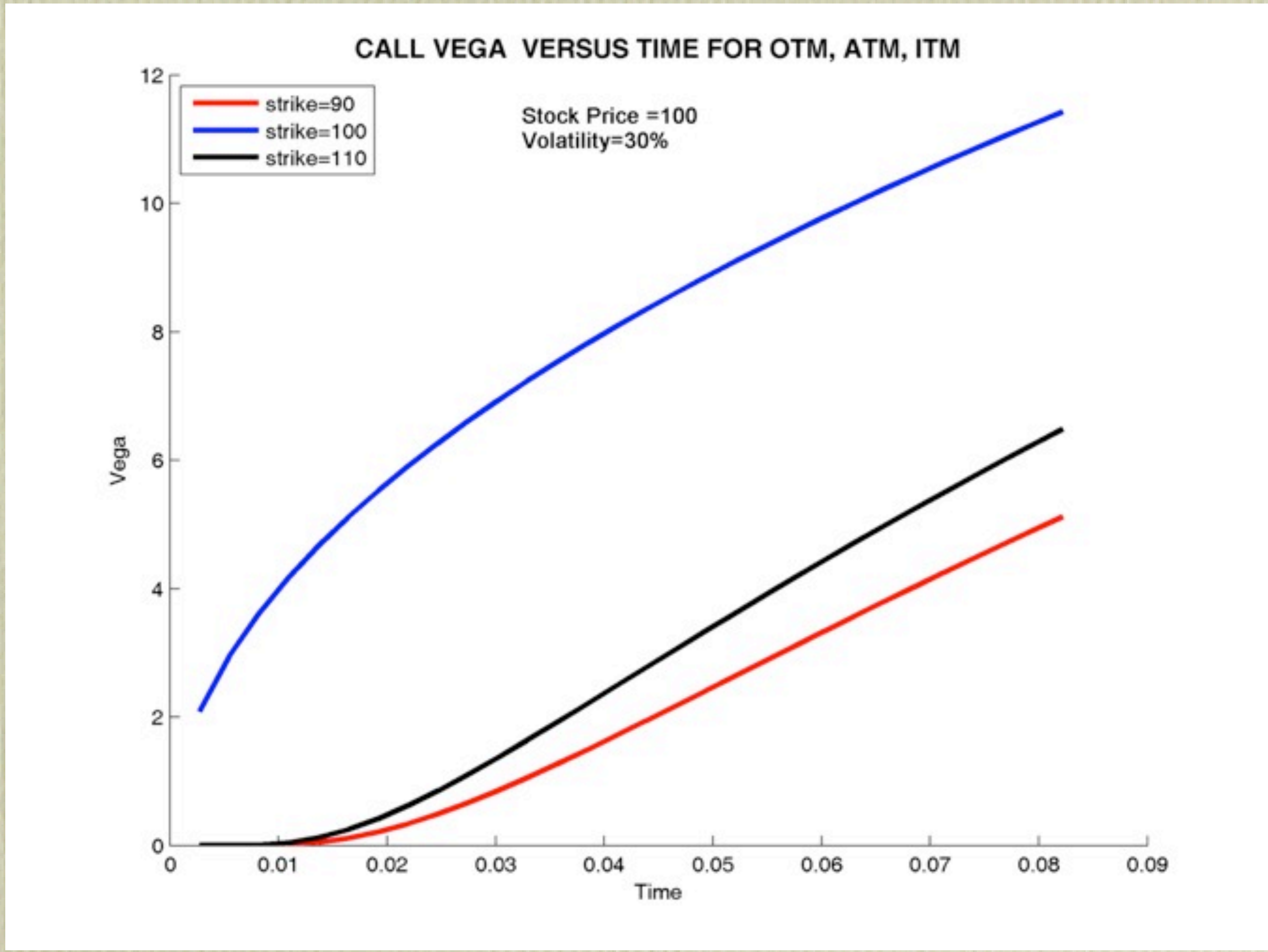
- Vega is the partial derivative of an option with respect to the underlying's price.

$$Se^{-q\tau}\varphi(d_1)\sqrt{\tau} = Ke^{-r\tau}\varphi(d_2)\sqrt{\tau}$$

Vega vs Stock Price



Vega


CALL VEGA VERSUS TIME FOR OTM, ATM, ITM
strike=90
strike=100
strike=110
Stock Price =100
Volatility=30%
Vega
Time
0
2
4
6
8
10
12
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09

CHARM

Charm or Delta Decay is rate of change of delta with time

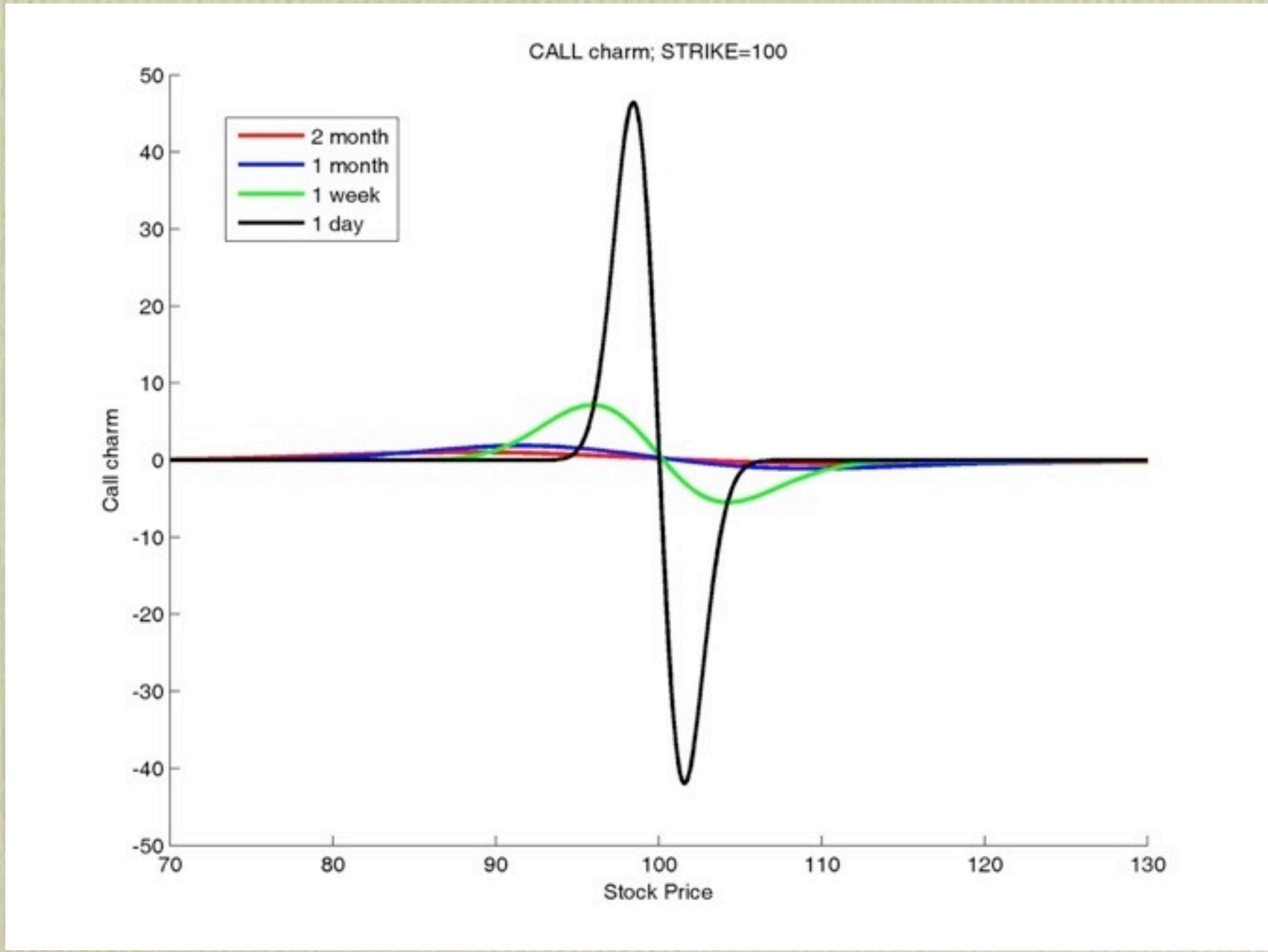
Call charm

$$-qe^{-q\tau}\Phi(d_1) + e^{-q\tau}\varphi(d_1)\frac{2(r-q)\tau - d_2\sigma\sqrt{\tau}}{2\tau\sigma\sqrt{\tau}}$$

Put charm

$$qe^{-q\tau}\Phi(-d_1) + e^{-q\tau}\varphi(d_1)\frac{2(r-q)\tau - d_2\sigma\sqrt{\tau}}{2\tau\sigma\sqrt{\tau}}$$

CHARM for a CALL

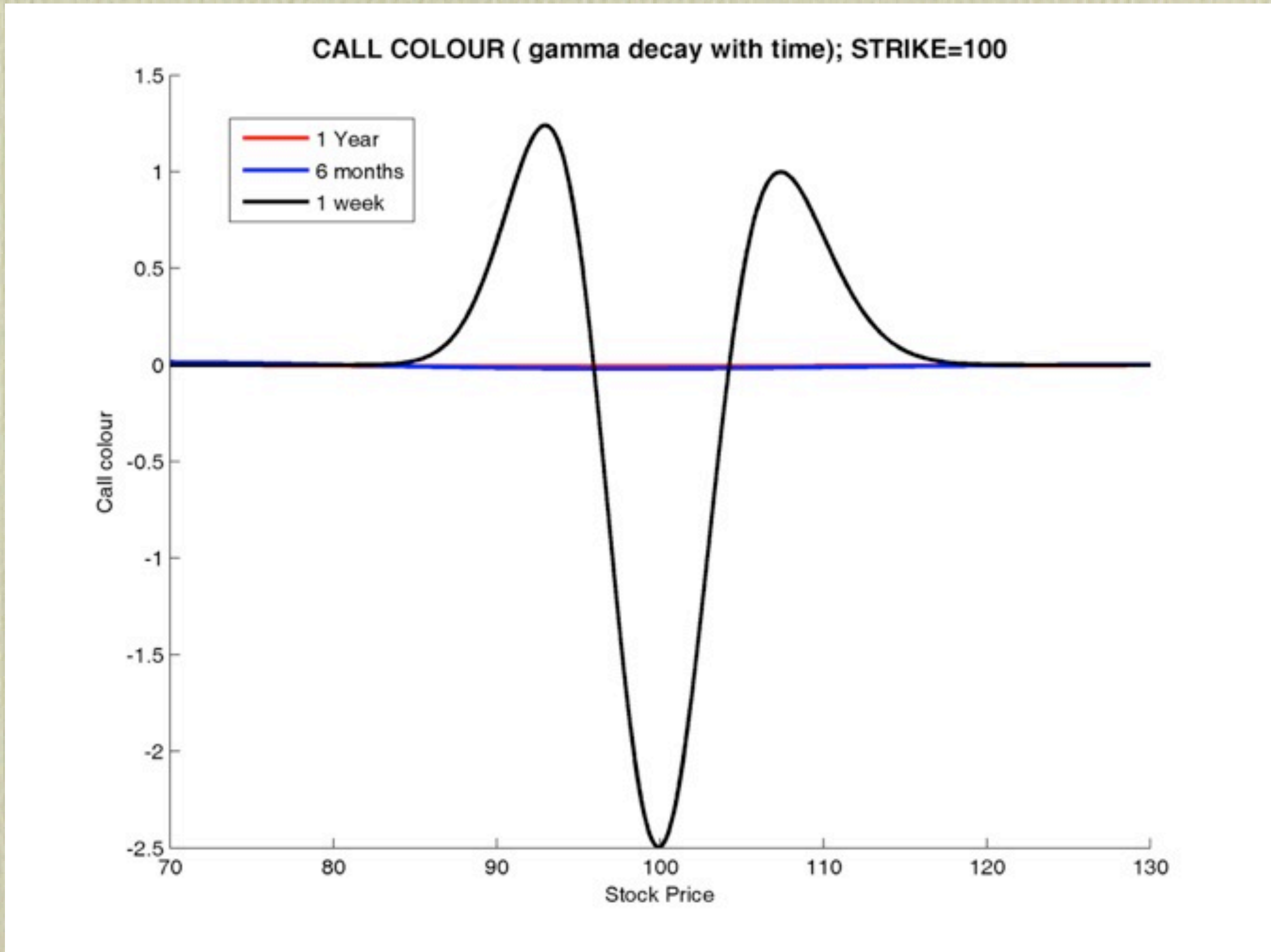


COLOR

Charm or Delta Decay is rate of change of delta with time

$$-e^{-q\tau}\frac{\varphi(d_1)}{2S\tau\sigma\sqrt{\tau}}\left[2q\tau+1+\frac{2(r-q)\tau-d_2\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}d_1\right]$$

Color vs S

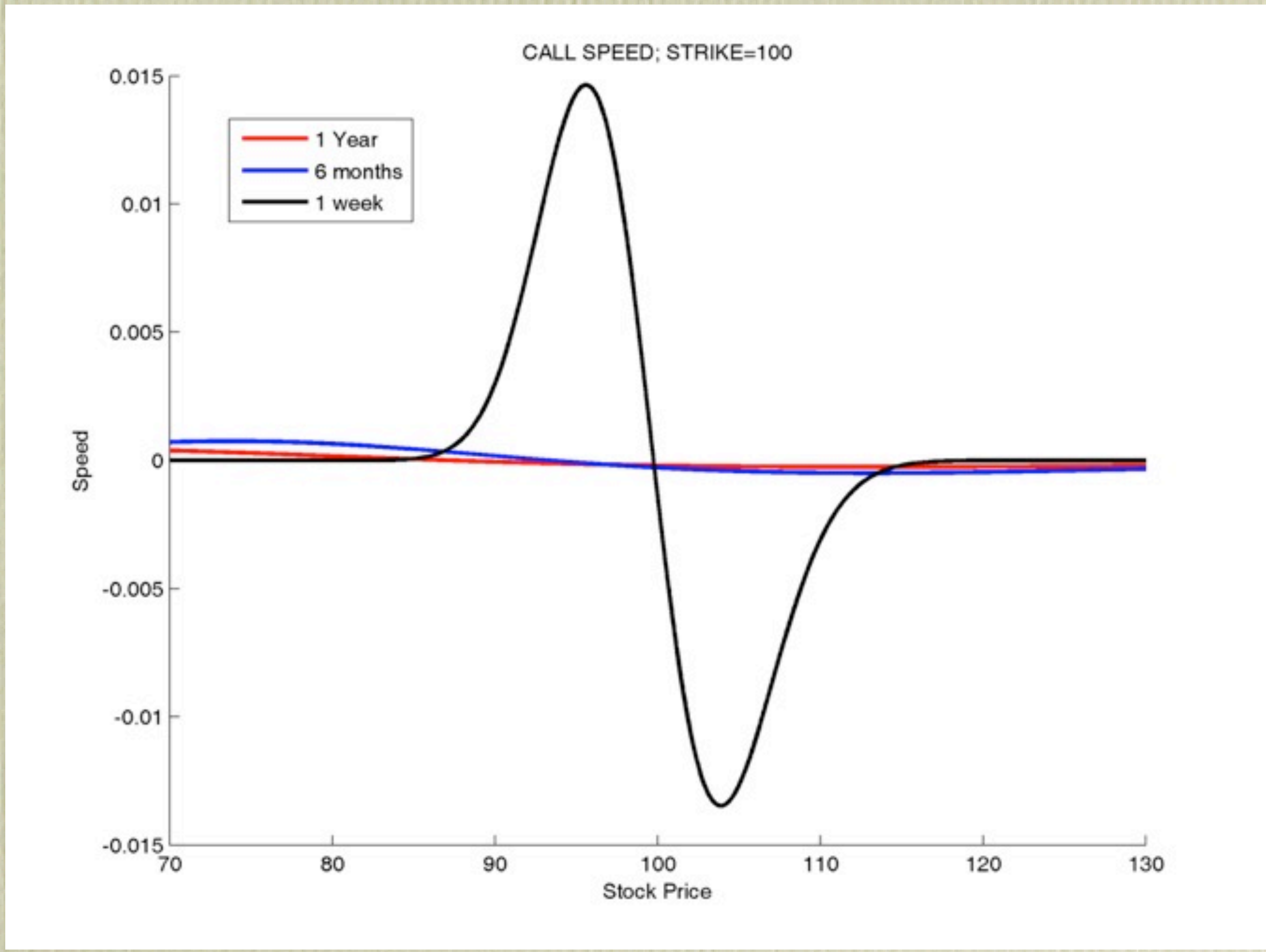

CALL COLOUR (gamma decay with time); STRIKE=100
1 Year
6 months
1 week
Call colour
Stock Price
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
70
80
90
100
110
120
130

SPEED

Speed or Gamma Decay is rate of change of gamma with the underlying.

$$-e^{-q\tau}\frac{\varphi(d_1)}{2S\tau\sigma\sqrt{\tau}}\left[2q\tau+1+\frac{2(r-q)\tau-d_2\sigma\sqrt{\tau}}{\sigma\sqrt{\tau}}d_1\right]$$

SPEED



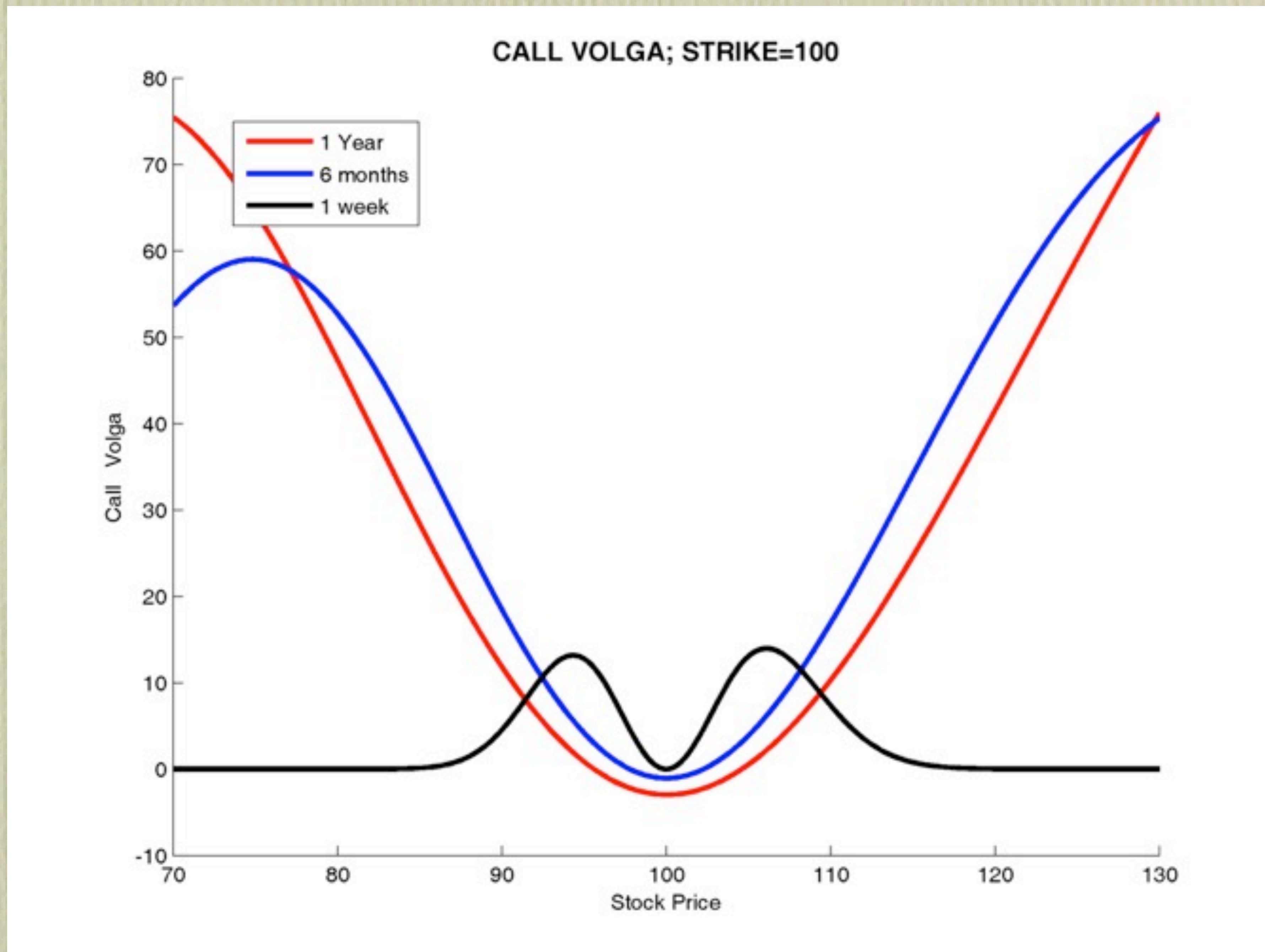
CALL SPEED; STRIKE=100
1 Year
6 months
1 week
Speed
Stock Price
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
70
80
90
100
110
120
130

VOLGA

Volga, or Vomma, is the Vega-gamma; the second order partial derivative of an option with respect to volatility.

$$Se^{-q\tau}\varphi(d_1)\sqrt{\tau}\frac{d_1 d_2}{\sigma} = \upsilon\frac{d_1 d_2}{\sigma}$$

VOLGA



CALL VOLGA; STRIKE=100
1 Year
6 months
1 week
Call Volga
Stock Price
80
70
60
50
40
30
20
10
0
-10
70
80
90
100
110
120
130

VANNA

Vanna is the partial derivative of Delta with respect to Volatility

$$-e^{-q\tau}\varphi(d_1)\frac{d_2}{\sigma} = \frac{\upsilon}{S}\left[1 - \frac{d_1}{\sigma\sqrt{\tau}}\right]$$

Taylor series expansion and P & L Attribution

Suppose you have a portfolio of options, Π
Assume it has options one one stock, S and that volatility for all the options is σ

$$d\Pi = \frac{\partial \Pi}{\partial y}\delta s + \frac{\partial \Pi}{\partial s}\delta t + \frac{1}{2}\frac{\partial^2 \Pi}{\partial s^2}\delta s^2 + \frac{1}{2}\frac{\partial^2 \Pi}{\partial t^2}\delta t^2 + \frac{\partial^2 \Pi}{\partial s \partial t}\delta s \delta t + ...$$

$$d\Pi = \Delta.\delta s + \theta\,\delta t + \frac{1}{2}\gamma\,\delta S^2 + ...$$

when terms of orders higher than δt are ignored.

If we assume that volatility can change,

$$d\Pi = \Delta.\delta s + \theta\,\delta t + \frac{1}{2}\gamma\,\delta s^2 + \kappa\,\delta\sigma + ...$$

Changes in portfolio values are attributed to the Δ, y, θ, k and the changes in S and t

How do Delta,Theta and Gamma relate to each other

Consider European call on a non-dividend paying stock.
we know that

$$\frac{\partial C}{\partial t} + rs\,\frac{\partial C}{\partial s} + \frac{1}{2}\,\frac{\sigma^2 s^2\,\partial^2 C}{\partial s^2} = rC$$

This is just the famous Black-Scholes-Merton equation.

since $\theta = \dfrac{\partial C}{\partial t}$, $\Delta = \dfrac{\partial C}{\partial s}$ $\gamma = \dfrac{\sigma^2 C}{\partial s^2}$

$$\theta + r\,s\,\Delta + \frac{1}{2}\sigma^2 s^2 \gamma = rC$$

If Δ is zero,you can see that θ and are in γ opposite directions.