Implied Volatility

Singapore Management University

QF 301

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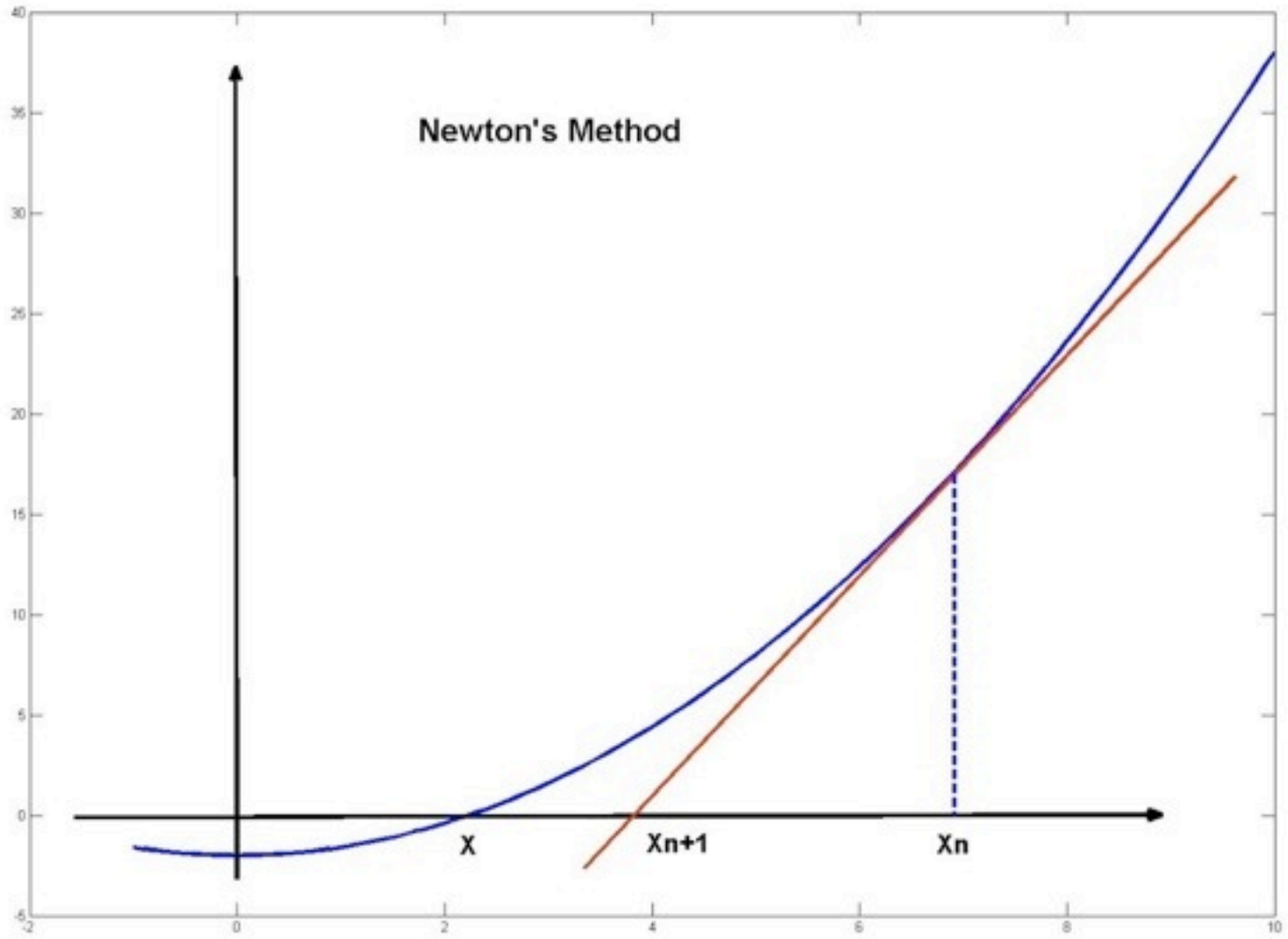
Implied Volatility

- What is it?
- How to calculate?
- Bisection Method and Newton-Raphson Method
- Smile
- Term structure

Bisection Method (Ms Nan's MATLAB code)

```
function midpoint = bisectionMethod(left, right, epsilon)

 %Bisection Method
while (abs(right - left) > 2*epsilon)

    %Calculate midpoint of domain
    midpoint = (right + left) / 2;

    %Find f(midpoint)
    if ((f(left) * f(midpoint)) < 0)

        %Throw away right half
        right = midpoint;

    elseif ((f(right) * f(midpoint)) < 0)

        %Throw away left half
        left = midpoint;

    else
        %Our midpoint is exactly on the root
        break;
    end
end

end
```

Newton's method



$$x_1 = x_0 - \frac{f(x_0)}{\frac{\partial f(x_0)}{\partial x}}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{\frac{\partial f(x_i)}{\partial x}}$$

So we have

$$f(\sigma) = c_{OBS} - c_{BSM}(\sigma)$$

$$\sigma_{i+1} = \sigma_i - \frac{c_{OBS} - c_{BSM}(\sigma_i)}{\frac{\partial c_{BSM}(\sigma_i)}{\partial \sigma}}$$

Drop in Implied Vols

- Before an important political announcement, like vote on a no-confidence motion, budget announcement, FOMC meeting, trading grinds to a halt. “Actual volatility” is low. But implied vols are high. After the event, vols drop as if a ballon that is punctured.

- Calls expensive before earnings!

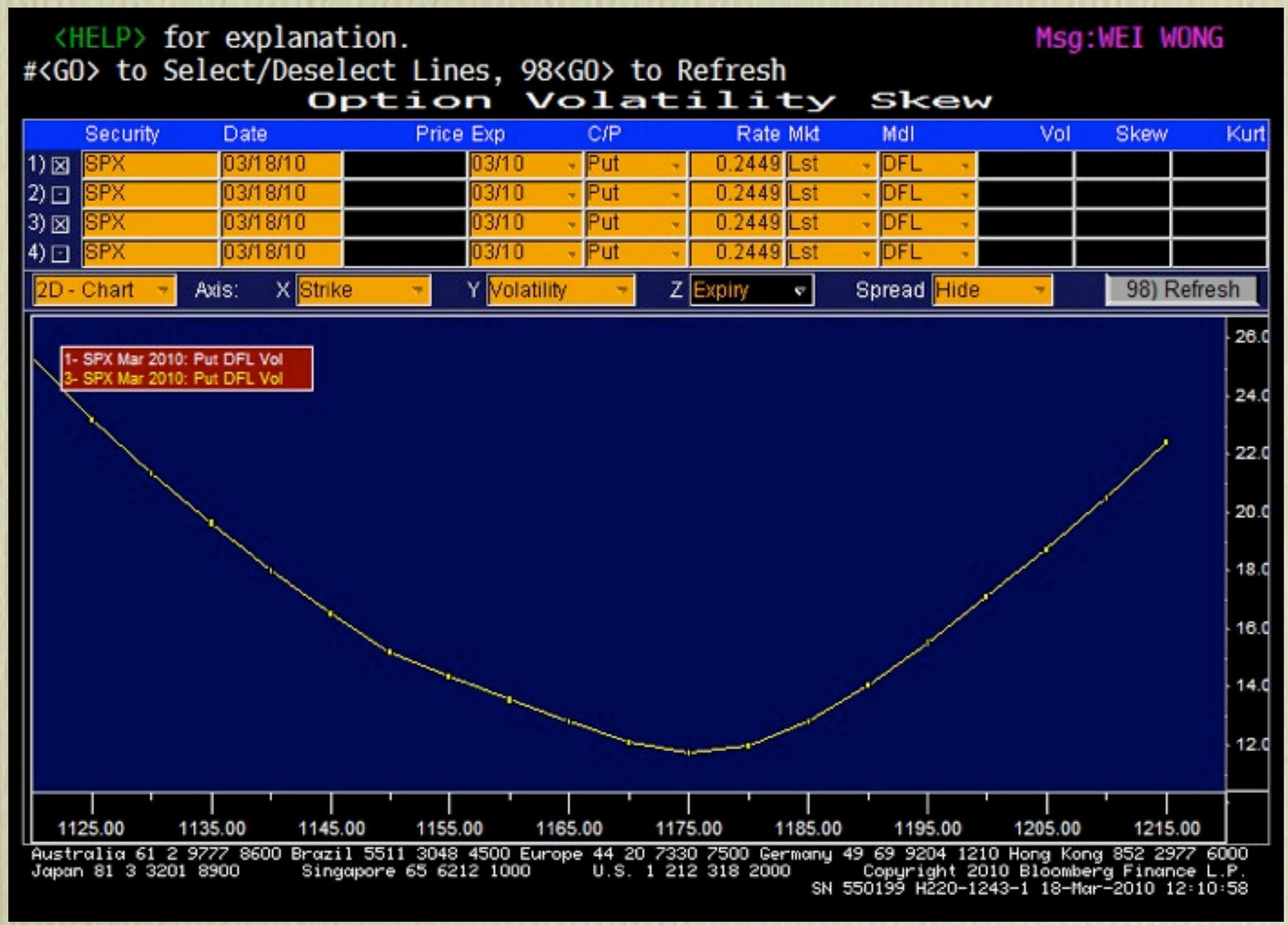
- Weekend effect: cautious of holding options over the weekend owing to theta. Therefore option prices drop and people adjust vols lower. of Monday

Rule of Thumb for ATM Option Price

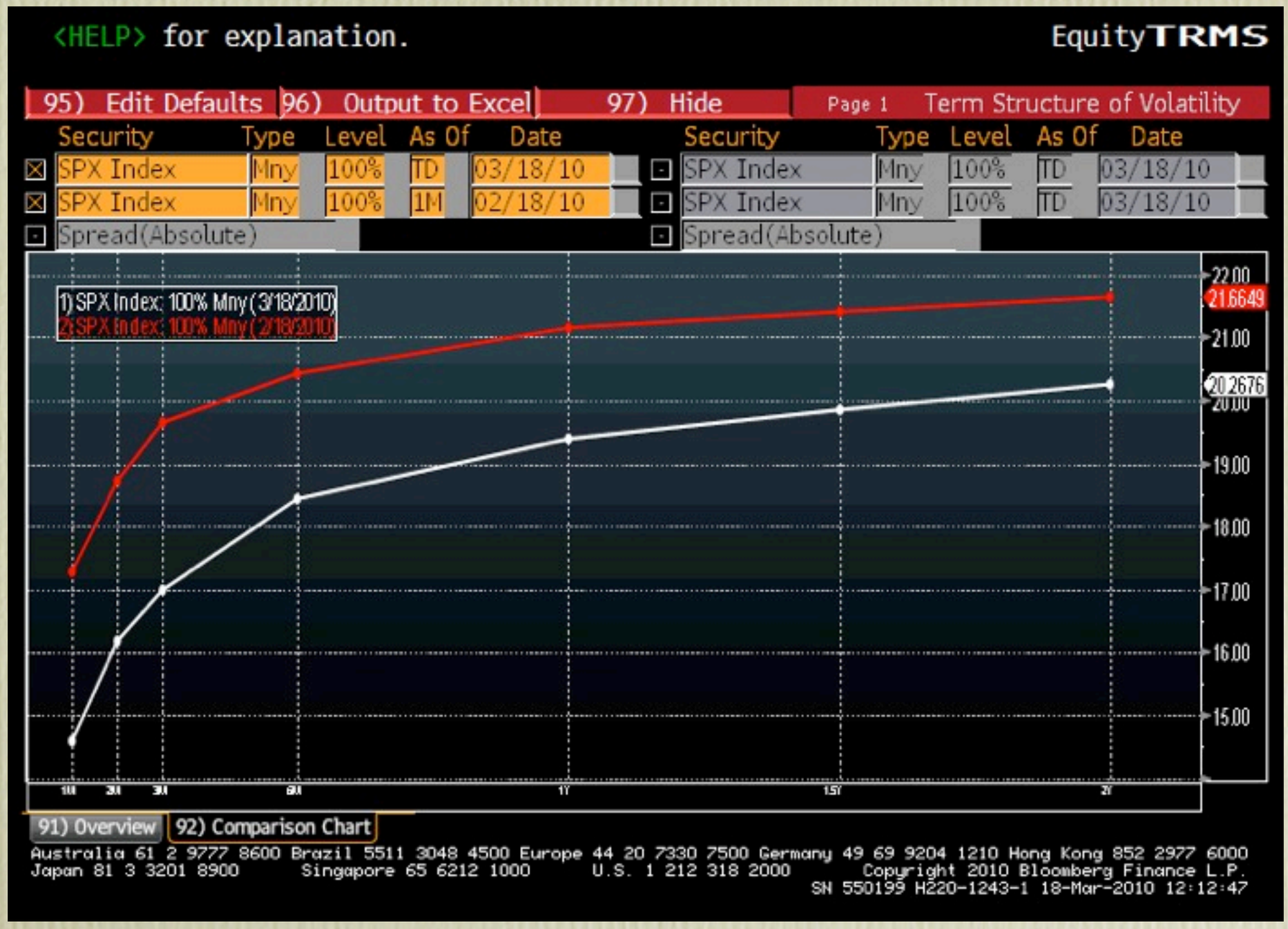
$$ATM\ Option = \frac{\sigma\sqrt{T}}{\sqrt{2\pi}}$$

$$and \sqrt{1/2\pi} = 0.3989$$

Volatility Smile for SPX options (SKEW on Bloomberg)



Term Structure of Volatility, SPX options (TRMS on Bloomberg)



Volatility Quotation in FX Derivatives

- Fx Markets use sticky-Delta to convey smile informations (Implied Volatility is quoted in delta rather than strike, e.g., 25 delta strike). If FX spot moves, the implied volatilities vs Delta stay the same but they change vs strike.

- Implied volatility quotes are given in terms of a straddle, risk reversal and butterfly (which is [strangle-straddle]/2)

To: SAURABH SINGAL (INDEA CAPITAL PTE LT)
At: 3/19 12:25:13

Hi Saurabh,

1-mth USDJPY ATM vols at 10%
1-mth USDJPY 25 delta vols is 10.4%
1-mth USDJPY 10 delta vols is 11.2%

ATM vols is 10%
25 delta risk reversal si 0.4% mid
25 delta butterfly is 0.25%

Allow forwarding? Y Delay message until mm/dd hh:mm Freq

Vanna-Volga Method

$$
\begin{aligned}
\text{ATM} &= \frac{1}{2}\text{Straddle}(K_{\text{ATM}}) \\
\text{RR} &= \text{Call}(K_c, \sigma(K_c)) - \text{Put}(K_p, \sigma(K_p)) \\
\text{BF} &= \frac{1}{2}\text{Strangle}(K_c, K_p) - \frac{1}{2}\text{Straddle}(K_{\text{ATM}})
\end{aligned}
$$

$$
X^{\text{VV}} = X^{\text{BS}} + \underbrace{\frac{\text{Vanna}(X)}{\text{Vanna}(\text{RR})}}_{w_{\text{RR}}} \text{RR}_{\text{cost}} + \underbrace{\frac{\text{Volga}(X)}{\text{Volga}(\text{BF})}}_{w_{\text{BF}}} \text{BF}_{\text{cost}}
$$

$$
\begin{aligned}
\text{RR}_{\text{cost}} &= [\text{Call}(K_c, \sigma(K_c)) - \text{Put}(K_p, \sigma(K_p))] - [\text{Call}(K_c, \sigma_{\text{ATM}}) - \text{Put}(K_p, \sigma_{\text{ATM}})] \\
\text{BF}_{\text{cost}} &= \frac{1}{2}[\text{Call}(K_c, \sigma(K_c)) + \text{Put}(K_p, \sigma(K_p))] \\
&\quad -\frac{1}{2}[\text{Call}(K_c, \sigma_{\text{ATM}}) + \text{Put}(K_p, \sigma_{\text{ATM}})]
\end{aligned}
$$

VIX

- The VIX or Volatility Index is a model free estimate of expected 30 day volatility of S&P500 options.
- Often moves opposite to equity markets and is sharply mean-reverting.
- Details on CBOE's website (http://www.cboe.com/micro/vix/vixwhite.pdf)
- The main thing from this paper is how the vix is calculated, shown in next slide.

$$VIX/100 \Rightarrow \text{VIX} = \sigma \times 100 \qquad \sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$$

σ is	$VIX/100 \Rightarrow \text{VIX} = \sigma \times 100$
T	Time to expiration
F	Forward index level derived from index option prices
K_0	First strike below the forward index level, F
K_i	Strike price of i^{th} out-of-the-money option; a call if $K_i > K_0$ and a put if $K_i < K_0$; both put and call if $K_i = K_0$.
ΔK_i	Interval between strike prices – half the difference between the strike on either side of K_i: $\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$
R	Risk-free interest rate to expiration
$Q(K_i)$	The midpoint of the bid-ask spread for each option with strike K_i

VIX Index


VIX 16.62Y as of close 3/18
Index GPC
VIX INDEX
Advanced
Hide
GPC - Candle Chart
Page 1/42
Range 01/07/00 - 03/18/10
Upper Candle Chart
Mov. Avgs 50 100 200
Currency USD
Period Weekly
Lower None
Mov. Avg 15
Events
Last 16.62
High on 10/24/08 89.53
Average 21.89
Low on 12/15/06 9.39
SMAVG on Close(50) 25.2670
SMAVG on Close(100) 31.8033
SMAVG on Close(200) 24.5677
90
80
70
60
50
40
31.80
25.27
20
16.62
10
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2010 Bloomberg Finance L.P.
SN 697146 H369-554-0 19-Mar-2010 17:15:13

Other VIX-like Indices

World Volatility Indices		
1) Americas	Last Price	Change on Day
3) AMEX QQQ VOLATILITY-QQV	16.50	--
4) CBOE DJIA VOLATILITY-VXD	16.42	1.08
5) CBOE NDX VOLATILITY-VXN	18.82	.81
6) CBOE OEX VOLATILITY-VXO	17.64	1.05
7) CBOE SPX VOLATILITY-VIX	18.40	.85
8) CBOE RTY VOLATILITY-RVX	23.81	1.53
9) Mexico	19.42	.00
2) Europe		
10) GERM VDAX VOLATILITY	16.39	-.68
11) FTSE100 Volatility	15.74	+.21
12) VDAX - NEW	17.76	+.00
13) VSTOXX	19.91	+.02
14) VSMI	14.29	-.14
15) VAEX VOLATILITY	18.85	+.15
16) VCAC VOLATILITY	17.79	+.12
17) VBEL VOLATILITY	15.15	+.21
3) Asia/Pacific		
18) India	17.05	-1.08
19) KOSPI200 VOLATILITY INDX	17.20	-.19

VIX like indices on OIL, Gold,Corn...